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GLACIAL GEOLOGY OF THE SNOQUALMIE-CEDAR AREA, WASHINGTON

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ABSTRACT

The Snoqualmie-Cedar area is a typical part of the border zone between the Cascade Range and the Puget lowland. The question at issue is the relation between Pleistocene alpine glaciers of the Cascades and the Puget lowland glacier. Evidence shows that the local Cascade glaciers were small and did not contribute to the Puget Glacier at any time during the last glacial stage. At its maximum the Puget Glacier pushed up into the Cascades, impounding long "Glen Roy" lakes in mountain valleys, some of which had been occupied shortly before by alpine glaciers.

INTRODUCTION

The Snoqualmie and Cedar rivers rise in the axial highlands of the Cascade Range and flow westward, issuing from the mountains into the Puget lowland approximately 100 miles south of the Canadian boundary. The Snoqualmie-Cedar area, as here defined, includes the lower mountain valleys of the Cedar River and the Middle and South forks of the Snoqualmie and contiguous parts of the Puget lowland. This area is treated here as a local unit, complete in itself; the regional problem of which it is a part is the relation between the Puget Glacier, which occupied the lowland during the Pleistocene, and local Pleistocene glaciers of western Cascade valleys. The paragraphs below are intended to explain the regional setting of the Snoqualmie-Cedar unit and to indicate the place of the present paper in a program of glacial study embracing the Puget region as a whole.

The earliest comprehensive reports on the glacial geology of the Puget region were published by Bailey Willis in 1897 and 1899,[1] on the basis of detailed studies in the Tacoma quadrangle and reconnaissance throughout the remainder of the lowland. He regarded the Vashon (Wisconsin) Puget lobe as a great piedmont glacier formed by the coalescence of ice streams from Canadian highlands to the north, the Olympic Mountains to the west, and the Cascade Range. Willis' theory, stated in regional terms, necessarily applies to the Snoqualmie-Cedar area, but his conclusions regarding large-scale contributions by local Cascade valley glaciers to Puget lowland ice were based primarily on studies farther south along the mountain front, and his papers contain no special mention of the area here described.

In 1913[2] J Harlen Bretz published a much more detailed account of the glaciation of the Puget region, holding that the Puget lobe was essentially of Canadian derivation and presenting evidence to prove that the lowland ice moved into the lower reaches of some valleys in the Olympic Mountains. He devoted little space to discussion of the eastern margin of the lobe, minimizing, without denying, the contributions of Cascade valley glaciers to the main mass. In this connection he states: "No evidence has been found to show that the Puget Sound Glacier crowded back into the Cascade Mountains as it did in the Olympics. Such evidence is not to be expected, since the glaciers of the Cascade Mountains were of much greater mass than those from the smaller Olympic Mountains."[3]

In a brief section on the Snoqualmie-Cedar area Bretz ascribed a supposed moraine near Snoqualmie Falls, at the mountain front, to a local Cascade glacier and appears to have associated with it the derangement of the Snoqualmie drainage that produced the falls. The great morainal embankments within the Snoqualmie and Cedar

[1] "Drift Phenomena of the Puget Sound," *Bull. Geol. Soc. Amer.*, Vol. IX (1898), pp. 111–62; Willis and G. O. Smith, "Tacoma Folio," *U.S. Geol. Surv. Atlas No. 54* (1899).

[2] "Glaciation of the Puget Sound Region, Washington," *Wash. Geol. Surv. Bull.* Vol. VIII (1913).

[3] *Ibid.*, p. 225.

valleys to be discussed below were regarded by him as remnants of an extensive drift-filling of pre-Vashon age; but no evidence was adduced in support of this age assignment, and the derivation of the drift, whether local or northern, was not discussed.

The writer's work along the eastern margin of the Puget lobe, begun in 1936 and continued at intervals since that time, has included study of the lower mountain portions of seven major stream valleys and the intervening Cascade mountain front; his students have reported briefly on one valley.[4] Remapping of the drift borders of the lobe around its southern and western (Olympic) margins is now in progress. This study, supported by a grant from the Penrose Fund of the Geological Society of America, is an extension of the earlier investigations along the eastern margin and will be reported in a paper on the marginal relations of the Puget lobe as a whole, including the Cascade front. Since adequate presentation of the mass of evidence bearing on Cascade-Puget ice relationships would unduly lengthen treatment of the regional aspects of Puget glaciation, it seems desirable to set forth separately and in some detail the evidence bearing on this particular issue in one typical segment of the Cascade front.

The Snoqualmie-Cedar area has been chosen because it is, in certain respects, ideally adapted for demonstration of essential relationships. Its central feature is a broad trunk valley, which, because it carries Puget lowland topography into the mountains, is here called the "Snoqualmie embayment" (Fig. 1). Three individual valleys which converge to form the embayment are similar in size and appear to have had closely similar histories during the last glacial stage. Postglacial stream-cutting has, however, been markedly different in the three valleys; in one the initial topographic form of the drift-filling is perfectly preserved, while deep trenching in the other two reveals its internal structure. For this reason there is a far more complete body of evidence available for the solution of glacial problems in the Cedar-Snoqualmie area, considered as a whole, than in any other single valley along the densely forested Cascade front.

[4] Allen S. Cary and Charles W. Carlston, "Notes on Vashon Stage Glaciation of the South Fork of the Skykomish River Valley, Washington," *Northwest Science*, Vol. XI (1937), pp. 61–62.

FIG. 1.—Topographic map of the Snoqualmie-Cedar area, redrawn from Sultan and Cedar Lake quadrangles, U.S. Geological Survey. Location of places and features mentioned in text is indicated by initial letters in margin: *SF*, Snoqualmie Falls; *S*, Snoqualmie (town); *NB*, North Bend (town); *MFE*, Middle Fork embankment; *SFE*, South Fork embankment; *CE*, Cedar embankment; *CM*, Camp Mason; The northwest-trending valley occupied by Cedar Lake is the Cedar Valley; the southwest-trending valley occupied by the Cedar River is referred to in the text as the "Cedar Spillway." Lakes Calligan (*LC*) and Hancock (*LH*) lie in small valleys in the mountain front in northern part of the map. Arrows indicate dip of foresets in the Tokul Creek delta near Snoqualmie Falls.

ORIGIN OF THE SNOQUALMIE-CEDAR EMBANKMENTS

GENERAL STATEMENT

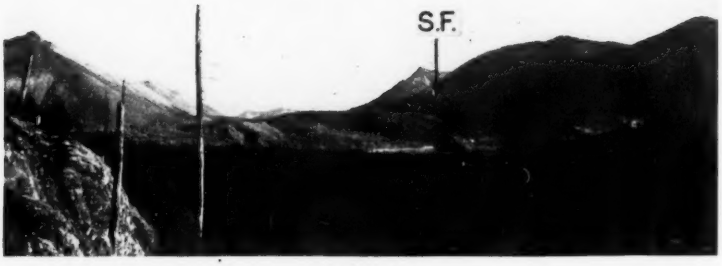
From a vantage point on the floor of the Snoqualmie embayment the observer may look northeastward, southeastward, and southward into the valleys of the Middle and South forks of the Snoqualmie and the Cedar rivers, respectively (Fig. 1). Mature lateral slopes of the three valleys, thus seen in profile, descend in normal fashion from the intervalley spurs, but they do not join normally in the axes of the valleys; at approximately the same level the lateral slopes of all three valleys are interrupted by flat-topped, hummocky ridges which appear to extend completely across the valley mouths (Fig. 2). These transverse embankments stand 700–900 feet above the general level of the floor of the Snoqualmie embayment and fall off toward it in steep, irregularly terraced slopes. The lack of harmony between the topography of the higher valley sides and that of the embankments suggests strongly that they are debris masses, postdating the valleys in which they lie. Excellent exposures in all three, showing contorted clay, irregularly bedded sands and gravels, and great boulders, prove that they are morainal. Evidence bearing on the derivation of the moraines—whether from local valley glaciers or from Puget Glacier—may be considered under two headings, namely, (1) the lithology of the materials of the embankments and (2) their topographic form.

LITHOLOGY OF THE EMBANKMENT MATERIALS

A large part of the drainage basin of the South Fork of the Snoqualmie is underlain by a uniform and distinctive granodiorite which constitutes the Snoqualmie batholith.[5] Local moraines in the headwater portions of the South Fork Valley are made up almost wholly of granodiorite; and numerous cliff exposures, traceable down-valley from known South Fork moraines to the vicinity of the valley-mouth embankment, give assurance that it would be similarly rich in this rock had the South Fork glacier had any part in its building. Cuts in the embankment reveal a great variety of rock types with Snoqualmie granodiorite notably rare (less than 1 per

[5] G. O. Smith and F. C. Calkins, "Snoqualmie Folio," *U.S. Geol. Surv. Atlas No. 139* (1906).

Middle Fork



South Fork

Cedar Valley

FIG. 2.—Views of the morainal embankments in the mouths of the valleys of the Middle and South forks of the Snoqualmie and the Cedar River taken from the Snoqualmie embayment. Initial letters and arrows mark the positions of the postglacial gorges of the streams.

cent). The most abundant single lithologic type, represented especially in large and angular blocks, is a metamorphosed basic igneous rock (hereafter referred to as "the Mount Si rock"), readily recognizable because of its greenish color and textural peculiarities, which is not known to occur in place in the South Fork drainage basin but which crops out in glaciated knobs in the Snoqualmie embayment and makes up a considerable part of the imposing mass of Mount Si. The position of Mount Si, at the angle between the mountain front and the Snoqualmie embayment, is such that its flanks would have been severely eroded by ice entering the embayment from the northwest. Its almost vertical west face, now streaming with talus slides, its scoured and plucked southern slopes, and the *roches moutonnées* of the embayment floor are the evident source of the boulders and blocks of Mount Si rock in the morainal embankment in the South Fork Valley.

The Middle Fork drains what appears to be a batholith border zone, composed largely of intrusives and metamorphics; a few tributaries on the west side of the valley just above its mouth head in the Mount Si complex. The wide variety of rock types available up-valley thus makes it impracticable to use any single lithologic type, qualitatively, in determining the direction of derivation of the morainal embankment which lies in the mouth of the Middle Fork Valley. But the abundance of Mount Si rock in the Middle Fork embankment and notable differences between suites of pebbles and ledge samples collected up-valley from those in the embankment indicate that this feature must have been formed by a glacier which moved into the valley mouth from the Snoqualmie embayment rather than by a local Middle Fork valley glacier. This view is strongly supported by the essential identity in lithologic composition of the Middle Fork and South Fork embankments.

The Cedar River drainage basin is underlain by the Keechelus andesite and associated sediments, with some intrusive granodiorite in the headwater area; no outcrops of Mount Si rock have been observed, and the general geologic relations are unfavorable to its occurrence. Richard E. Fuller,[6] in mapping the bedrock geology of the

[6] "Geology of the Northeastern Part of the Cedar Lake Quadrangle, Washington," Thesis No. 1201 (1925), University of Washington Library.

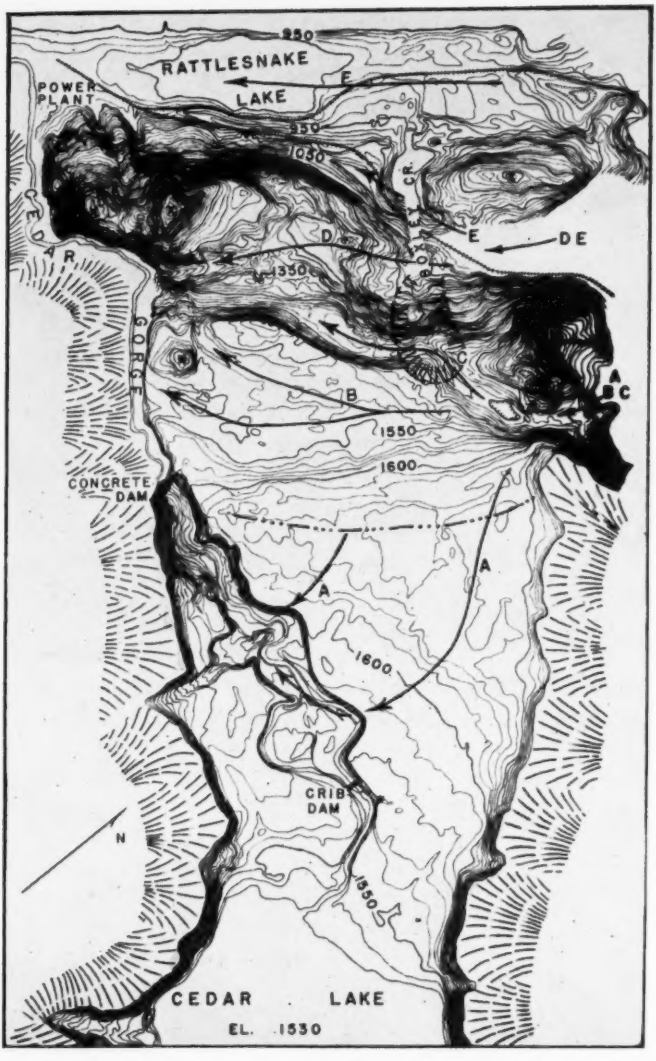


FIG. 3.—Map of the Cedar embankment (redrawn from map on file in Seattle engineer's office). Contour interval 10 feet. Contoured hills marked × are bedrock. The axis of the filled, preglacial valley of the Cedar River probably lies midway between hachured mountain slopes; the thickness of the fill may exceed 800 feet. Dash-dot line indicates crest of the morainal embankment. Note contrast between gentle up-valley slope from the crest toward Cedar Lake and the steep, terraced north face of the embankment. Arrows mark routes of meltwater at maximum (*A*) and recessional stands (*B*, *C*, *D*, *E*) of the Puget Glacier. Boxley amphitheater is indicated by hachures superposed on contours which show surface form before the outburst occurred.

Snoqualmie-Cedar area, was impressed by the abundance of boulders of Mount Si lithologic types in the Cedar morainal embankment and reconciled this observation with the views of his predecessors regarding glacial movements in the district by postulating the existence of an isolated mass of Mount Si rock underneath the waters of Cedar Lake. He now favors the view advanced here, namely, that the

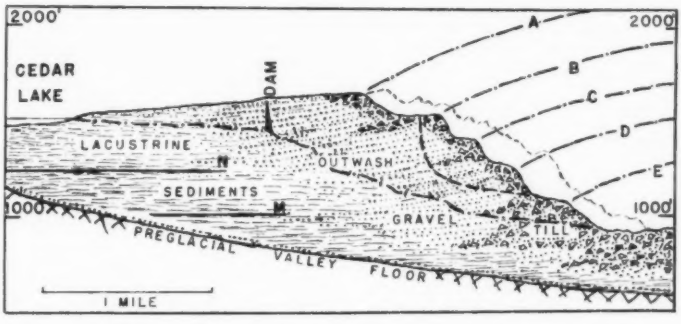


FIG. 4.—Diagrammatic and hypothetical structure section of the Cedar embankment along axis of preglacial valley. Outwash gravels deposited during advance of the Puget Glacier overlap and grade southward into lacustrine sediments and are in turn overlapped by till. At an early stage of ice advance Cedar Lake occupied the position of the solid line *M*, at a later stage the position *N*, and so on to its present position. At all stages of advance and at the maximum stand of the glacier (dash-dot line *A*) meltwaters flowed southward into Cedar Lake (see meltwater routes *A*, Fig. 3). Dash-dot lines *B*, *C*, *D*, and *E* suggest recessional stands of the ice during which the till on the north face of the embankment was thinned by meltwater scour (compare with terraces and meltwater routes *B*, *C*, *D*, and *E*, Fig. 3). Heavy dash line indicates approximate outline of Boxley amphitheater. Dash-dot line marks profile of postglacial Cedar gorge projected into the plane of the structure section. Depth and slope of the preglacial valley floor are unknown.

Cedar moraine was built by a glacier advancing up the valley from the north rather than down the valley.

In brief, then, the three valleys which converge in the Snoqualmie embayment differ markedly in bedrock geology, but the moraines which cross their mouths are so similar in lithology as to be indistinguishable, each from the others. The Mount Si rock, which, because of its limited outcrop area and distinctive character, serves well as an index stone, is common in all three morainal embankments and

indicates that they were formed at the terminus of a tongue of the Puget Glacier which advanced eastward into the Snoqualmie embayment.

TOPOGRAPHIC FORM OF THE EMBANKMENT

As pointed out in the introduction, the morainal embankments have been modified by postglacial stream-cutting in different degrees in each of three valleys; the one in the Cedar Valley has been little changed from its initial form and serves as a type. Had this moraine been built by the local Cedar valley glacier adjacent to an ice-free Snoqualmie embayment, it should have been flanked on the north by a northward-sloping outwash plain. The north face of the Cedar embankment is, however, a 700-foot scarp which descends steeply to the Snoqualmie embayment. Since the knobby, kettle-pitted morainal surface extending northward from the base of the scarp and the topographic details of the scarp itself could not have been formed by postglacial stream-swinging, this feature must be regarded as an ice-contact face. Complementary proof that the moraine was built by ice from the north is a long, smooth surface, sloping southward from the crest of the embankment to Cedar Lake (Figs. 3 and 4). Both the direction of slope of this surface and the current bedding and lithology of the gravels of which it is composed indicate that it is an outwash apron, spread *into* the valley by meltwaters from the north and northeast.

The scarps descending to the Snoqualmie embayment from the Middle and the South Fork moraines are similar in topographic form to that of the Cedar embankment and are, like it, ice-contact slopes essentially unmodified by postglacial erosion. The broad plateau of drift in the Middle Fork Valley is highest and somewhat hummocky along its southwestern edge, and its surface slopes northeastward (up-valley) for over a mile before breaking off in steep swing scarps that descend to the present stream (Fig. 1). It is, therefore, analogous to the up-valley-sloping Cedar outwash apron. No extended remnants of outwash-plain surface are preserved in the South Fork Valley, where the embankment has been reduced to a narrow ridge by stream cutting on the east (Fig. 1).

The embankments have been treated above as individual topographic features. Considered together, all three constitute one major

morainal unit, semicircular in plan and convex eastward. This alignment of the morainal embankments could be explained only by co-

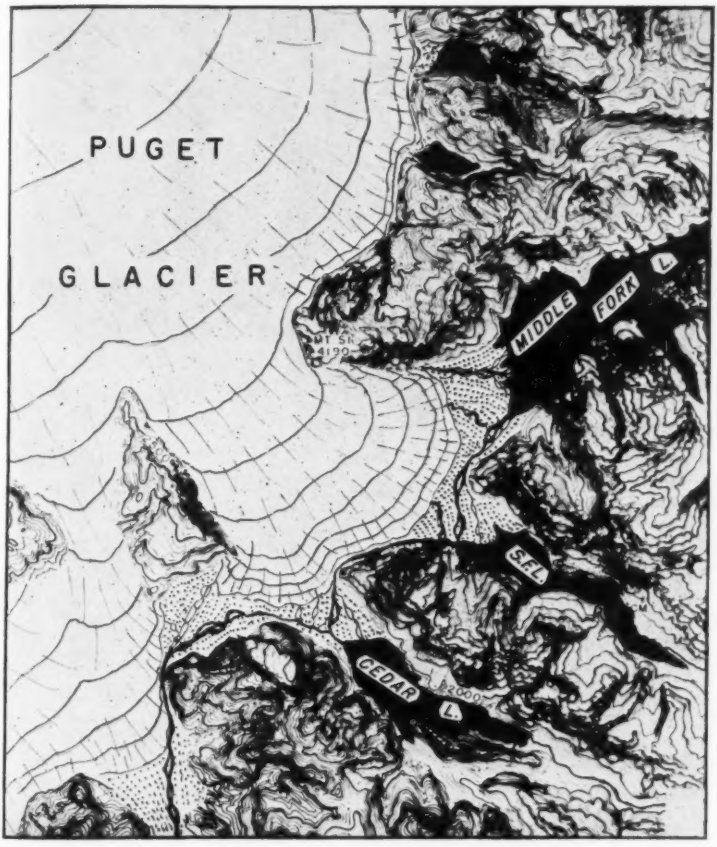


FIG. 5.—Maximum stand of the Vashon Puget Glacier (Embankment substage). Meltwaters from the glacier built deltaic outwash plains into the Middle Fork, South Fork, and Cedar lakes, and discharged from them in ice marginal channels (see text for qualifications). Local glaciers which doubtless occupied cirques in the mountains are not shown.

incidence had they been formed by three independent valley glaciers; the semicircle of ice-contact slopes is, according to the theory pro-

posed here, the mold of the Snoqualmie embayment lobe of the Puget Glacier (Fig. 5).

A

B

FIG. 6.—Views in the Snoqualmie Valley. *A*. View of the south flank of Mount Si, showing contrast between ice-scoured and moraine-ridged lower slopes and the upper unglaciated slopes. *B*. Lacustrine clay in the South Fork Valley, ¼ mile east of Camp Mason. Inset shows detail of bedding, the light-colored layers being clay and the darker layers silt. The highest varve is 14 inches thick.

Supplementary evidence bearing on the origin of the Snoqualmie-Cedar morainal embankments includes: (1) strongly developed crag

and tail topography in a foothill belt along the western base of the Cascades, indicating a direction of movement of Puget ice directly toward the mouth of the Snoqualmie embayment; (2) *roches moutonnées* with contrasted stoss and lee slopes on the south flanks of Mount Si and on the embayment floor, which indicate the same movement direction well within the mountain front; and (3) a systematic eastward slope of the Puget ice surface, as marked by the height of the fresh exotic pebbles, fresh glaciated ledges, and moraines along the mountain front and on the sides of the embayment (Figs. 5 and 6, *A*). These features require no extended verbal description.

RELATIONS OF THE EMBANKMENTS TO LAKES AND LACUSTRINE SEDIMENTS UP-VALLEY

For a distance of 5 miles up-valley from its embankment, bedded clays and silts are exposed in stream cuts and road cuts in the valley of the South Fork of the Snoqualmie. The clays range in elevation from 1,000 feet at the down-valley end of the exposures to 1,600 feet at the up-valley end, but the difference between these figures does not represent the total thickness, for the base of the down-valley clays is below the present river level. The greater part of the sequence is merely laminated, but some cuts show systematic alterations of thick silt layers with clay layers which are interpreted as varves. In one exposure in the lower part of the sequence individual varves average 10 inches in thickness, about twenty-five being recorded in a bank 20 feet in height (Fig. 6, *B*).

The South Fork has cut a postglacial valley approximately 600 feet deep through its valley-mouth embankment and is now caught on ledges of the preglacial south valley side, in which it is incising a slotlike gorge with waterfalls (Fig. 7). In the natural section of the embankment exposed in the sides of the transverse valley, lacustrine clays and silts are interbedded with sands, gravels, and till sheets. Parts of the sequence show distortion by ice thrusting.

In these exposures deltaic foresets and current bedding in gravel, sand, and silt and the trend of gravelly channel fillings indicate that the direction of flow of the depositing waters was to the west and southwest (Fig. 7). This would seem to be precisely the reverse of what the relations described above should lead one to expect, since

the Puget Glacier, advancing from the west, should have discharged its meltwaters *eastward into* the South Fork Lake. But it must be remembered that meltwaters entering the South Fork Valley from ice blocking the valley mouth must, in the absence of other outlet, have escaped from the valley along a route marginal to the ice

FIG. 7.—View looking eastward through the narrow valley cut by the South Fork in the South Fork embankment. The preglacial South Fork valley lies to the north (*left*); the river is here cutting in bedrock of the preglacial south valley side and is incised in rock gorges immediately upstream and downstream (Twin Falls). Lacustrine clay, interbedded with gravel and till, appears in the upper part of the morainal bank to the left. Foreset bedding, dipping westward (*toward the camera*) is seen in the center just above the road.

blockade. It will be shown below that *at the maximum stage* of the Puget Glacier all glacial lake outlets in the Snoqualmie-Cedar area were between the entering ice tongues and the southern sides of the dammed valleys (Fig. 5). The South Fork embankment cuts, 600 feet below the level of the crest of the embankment, are immediately adjacent to the preglacial south valley side; the seemingly anomalous sedimentary structures indicate that the outlet of the South Fork Lake was, likewise, in a south marginal position at an *early stage* of ice advance and embankment-building. Similar westward-

dipping structures in glaciofluvial and glaciolacustrine materials at several higher levels on the south valley side indicate that this marginal position of the outlet was maintained as the level of the lake was raised progressively by increase in height of the glacial barrier.

The drainage of the South Fork Lake at its maximum level flowed southwestward into Cedar Lake, a distance of approximately 4 miles (Fig. 5). The position of the connecting stream, or series of ice-marginal ponds, is marked by immense sand and gravel deposits, with westward-dipping foreset bedding and current bedding, preserved as terrace forms and minor ravine fillings at a fairly uniform level of 1575–1625 feet against the south valley side. Sheer cliffs between the sand fills suggest that locally the interlake channel may have been floored by ice. The meltwaters entered the Cedar Valley through a saddle between the main Cedar-South Fork Snoqualmie divide and a minor outlying rock knob, well shown on the topographic map of the Cedar embankment (Fig. 3).

Additional evidence bearing on the height of the South Fork Lake is furnished by a number of fan deltas of side streams, built out into the lake near its eastern or up-valley end. The distal ends of several of these features stand at about 1,600 feet, the level noted above as the approximate elevation of the outlet channel.

Observations in the Middle Fork Valley repeat and, in certain respects, supplement those along the South Fork. Postglacial stream-swinging has, in this case, sectioned the *up-valley* face of the embankment, near the *center* of the preglacial valley. In these cuts bedded sands and gravels of the embankment grade up-valley into lucustrine clays, and short deltaic foresets are directed *up-valley*. These deposits were evidently formed at a point of inflow into the Middle Fork Lake, in contrast with those described in the South Fork Valley, which were formed at a point of outflow. Upstream from the embankment the whole valley floor is a jumble of landslides and flows in bedded clays. A typical undercut bank near the junction of Taylor River with the Middle Fork, 7 miles up-valley from the embankment, exposes over 100 feet of laminated or varved clays with the base below the present river level.

One of the most striking features of the Middle Fork Valley is a "shoreline," somewhat different in origin but reminiscent of the

famous Parallel Roads of Glen Roy[7] (Fig. 8). It is best seen from the air; on the ground it resolves itself into a series of tributary fan-delta apexes, connected by a faint change in slope and vegetation on stabilized talus slides. The uniform level of the shoreline, maintained for a distance of 2 miles in spite of local variations from fan to fan, together with the level of the distal end of the deltaic out-

FIG. 8.—Airplane view looking east-southeast across the valley of the Middle Fork of the Snoqualmie (*foreground*). The large valley on the right (*south*) is occupied by the South Fork. The water body in the distance is Lake Keechelus (*L.K.*), a moraine-dammed lake on the east flank of the Cascade Range. Note the glaciated form of the valleys which join the Middle Fork Valley; these tributary valleys have cirques and cirque lakes at their heads (see general map, Fig. 1). A straight edge, placed on the photograph at level of dash in left margin will lie along the "shoreline" described in the text; higher and lower markings on the mountain flanks are logging roads. The hills in the right foreground are part of the Middle Fork Valley mouth embankment. (Pacific Areal Surveys.)

wash plain built into the lake from the Puget Glacier, fixes the level of the Middle Fork water body at 1550–1600 feet. The outlet channel from this lake into the South Fork Lake is represented by a broad sag where the two embankments abut against the descending rock spur between the two valleys, but a growth of dense forest obscures topographic details.

Finally, in the Cedar Valley the Pleistocene lake still remains.

[7] T. F. Jamieson, "On the Parallel Roads of Glen Roy," *Quart. Jour. Geol. Soc.*, Vol. XIX (1863), p. 235.

Its relation to the Cedar morainal embankment is shown by the detailed contour map and structure section (Figs. 3 and 4).

The Snoqualmie-Cedar glacial lakes have been depicted in Figure 5 as separate water bodies joined by ice marginal streams. This concept must be qualified by a statement of the possibility, suggested by relations of the five Berg Lakes similarly impounded in mountain valleys by a lobe of the Bering Glacier,[8] that the drainage of the Snoqualmie-Cedar lakes may have been subglacial or englacial during some stages of ice occupancy. The overlapping in ranges in elevation given for the three lakes and the presence of thick deltaic foresets in the terrace along the steep mountain front between the Middle Fork and Cedar valleys suggests, in addition, the possibility that during some stages the three lakes may have been connected by open water, thus constituting one large, digitate water body. This condition would have obtained if the ice blockade at the final (southwestern) outlet of the Snoqualmie-Cedar area had been higher than those parts of the Puget Glacier which blocked the individual valley mouths.

ORIGIN OF THE CEDAR SPILLWAY

It is evident from topographic relations that the greater part of the drainage of the Snoqualmie-Cedar area escaped through the lower, southwestward-trending segment of the Cedar Valley (the Cedar spillway, Fig. 1) during the stages of glacial occupancy. This valley is now flanked on the south by a broad, southwestward-sloping terrace, locally pitted by shallow kettles (Fig. 9). The terrace surface coincides in elevation with the highest level of Cedar embankment outwash and is apparently a remnant of a valley train formed by meltwaters discharged from the Cedar-Snoqualmie area during the maximum stand of the Puget Glacier. The absence of bedrock exposures in its steep frontal scarp or in transecting tributary gorges proves that the terrace is part of a valley fill and that the cutting of the deep rock valley in which it lies must antedate the last glacial stage.

It seems probable that Cedar River originally flowed northwestward, parallel with the regional and local strike, as part of the

[8] See U.S. Geol. Surv., topographic map of the Controller Bay Region, Alaska.

Snoqualmie drainage system (Fig. 1). Invasion of the Snoqualmie-Cedar trunk valley by Puget ice during an earlier Pleistocene stage might then have caused ponding (as in the last stage) and diversion of meltwaters across a low place in the southwestern divide, possibly along a fault zone suggested by the alignment of the Middle Fork and the lower Cedar valleys. Incision of part or all of the Snoqualmie-Cedar drainage in the course so determined, during one or

FIG. 9.—View looking eastward across the Cedar spillway. The valley in the foreground is occupied by the southwestward-trending segment of the Cedar River (see Fig. 1). The terrace is a remnant of the fill formed by discharge from the Snoqualmie-Cedar area at the maximum stand of the Puget Glacier. The valleys in the background are, from left to right, the Middle Fork of the Snoqualmie, the South Fork of the Snoqualmie, the Cedar Valley, and, on the extreme right, the preglacial valley of the Green River. (U.S. Forest Service.)

more interglacial stages, could then have produced the deep rock valley utilized as a spillway during the last glacial invasion. In a number of similar situations elsewhere along the Cascade front this hypothesis is supported by direct evidence.

INTERNAL CONSTITUTION OF THE EMBANKMENTS

MECHANISM OF ORIGIN

Evidence has been presented tending to prove that the Snoqualmie-Cedar embankments are terminal moraines formed by a tongue of the Puget Glacier which blocked the mouths of the Snoqualmie and Cedar valleys and that lakes existed in these valleys

during the period of morainal accumulation. The accompanying diagrammatic cross section illustrates certain special structural and stratigraphic relations in the morainal fills which are the direct results of this mode of origin (Fig. 4). The section is drawn for the Cedar Valley, where the initial topographic form of the embankment is still largely preserved. It is based on study of surface exposures in the Cedar Valley, and, in addition, it embodies all available evidence in the valleys of the two forks of the Snoqualmie, where deep postglacial stream-trenching has laid bare parts of the glacial fill which are still hidden from direct observation in the Cedar Valley. The section is drawn along the axis of the filled valley and depicts relations of sediments deposited by meltwaters which flowed *into* the lake from the Puget Glacier. As pointed out in an earlier paragraph, significantly different conditions should be expected near the southern side of the filled valley, where the water-laid parts of the fill were deposited in part by an ice marginal stream flowing *out of* the lake.

It appears that a small lake was formed in the valley when the entering glacier first blocked its mouth. The lake was progressively increased in size, shifted up-valley, and raised in level as the ice advanced (Fig. 4, *M–N*). It was at all times floored by fine-textured sediments deposited from suspension, and it was usually separated from the ice front by a greater or less breadth of gravelly deltaic outwash plain. Especially during later stages of upbuilding of the embankment the advancing glacier accommodated itself to the growing barrier in its path by upthrust at the frontal edge.

A uniform rate of ice advance and proglacial deposition should, under these circumstances, have resulted in a progressive up-valley overlap of lake-bottom silts and clays by sandy and gravelly deltaic foresets, of these lacustrine foresets by thick stream-laid topsets, and of these outwash deposits by till, with more or less ice shoving recorded at the till-outwash contact. The actual complex interfingering of till, outwash gravels, and lacustrine sediments, seen especially in cuts in the Middle and South Fork valleys, and indicated in diagrammatic fashion in the cross section, is clearly due to oscillations in the position of the ice front, shifting in routes of meltwater streams, and variations in the level of the lakes.

SIZE OF THE EMBANKMENTS

Ice-contact slopes descending from the crests of the Snoqualmie-Cedar embankments vary in height from 500 to 700 feet; a conservative estimate makes the detrital fill in the axial parts of the preglacial valleys more than 800 feet thick. The embankments are, then, as morainal accumulations, immense by any standard of comparison; they exceed by many times the largest terminal moraines built by the main Puget Glacier in the southern part of the Puget lowland. The great size of the embankments is evidently due to the fact that they were built into lakes which acted as sediment traps, retaining not only the coarser fractions of the detritus outwashed from the glacier but a large share of the rock flour as well. The structure section indicates that the embankments are made up largely of water-laid materials and that till is merely a veneer of varying thickness of the ice-contact slopes.

FAILURE OF THE CEDAR RESERVOIR

The costly and disastrous failure of the Cedar Reservoir provides a striking confirmation of the theory of origin and constitution of the Cedar embankment outlined above. The history of the Cedar project has been published elsewhere; hence, only one aspect of the failure which bears on the present discussion need be mentioned here.[9]

In 1914 the City of Seattle completed a concrete dam in the postglacial Cedar River gorge, for the purpose of increasing the storage capacity of Cedar Lake (Figs. 3 and 4). As indicated in the structure section, the gorge above the dam is cut in open-textured outwash gravels which make up the bulk of the embankment. In 1918, after completion of certain sluicing operations intended to stop very serious leakage of water from the reservoir through the gravelly northeastern side of the gorge, the water was raised to the spillway (1,555 feet). Eight days after the water reached its full height an outburst occurred in the eastern part of the ice-contact slope 6,000 feet from the dam. The initial discharge, estimated at from 3,000 to 20,000 second-feet, washed out from 800,000 to 2,000,000 cubic yards of detritus in a few hours, producing a great amphitheater-shaped

[9] "A Geologic Interpretation of the Failure of the Cedar Reservoir," *Univ. Wash. Eng. Exper. Stat. Bull. 107* (1941).

crater. The flood passed down the valley of Boxley Creek to the Snoqualmie River, causing damage which resulted in a large judgment against the City of Seattle. The outflow quickly decreased to a few hundred second-feet and tapered off to a few second-feet as the water level in the reservoir decreased. There was no sudden lowering of the water in the reservoir at the time of the outburst.

The walls of the Boxley amphitheater are composed from bottom to top of gravels which are identical in lithology with the gravels outcropping in the gorge wall above the dam and which are surely a part of the same sequence of outwash deposits. Till is exposed at a number of points on the ice-contact slope east and west of the Boxley amphitheater and makes up the throat of the amphitheater. It appears that water moved laterally out of the reservoir into the open-textured gravels of the embankment, where it accumulated in great volume behind the relatively impervious till which veneers the ice-contact face. The immediate cause of the burst appears to have been a breaching of this till seal, which permitted the water impounded in the outwash gravels to escape with a rush. The Cedar project deserves a prominent place in the list of reservoir failures due to an almost complete disregard, by the builders, of the geology of the reservoir walls.

RELATIONS OF LOCAL CASCADE GLACIERS TO THE VASHON PUGET GLACIER

GENERAL STATEMENT

The Cascade Range, in the latitude of the Snoqualmie-Cedar area, shows every mark of severe alpine glaciation during the Pleistocene: all peaks above 6,000 feet approximate matterhorn forms, well-defined cirques occur as low as 3,000 feet, and, on the eastern flanks of the range, bulky moraines mark the termini of Cascade valley glaciers many miles in length. Numerous glaciers still exist on peaks along the axis of the range. Deductive reasoning, therefore, favors the view of Willis and Bretz, namely, that Cascade valley glaciers should have advanced at least to the western mountain front during the last glacial stage, which saw the whole Puget lowland flooded by ice of Canadian origin. Field relations indicate, however, that this was not the case in the Snoqualmie-Cedar area.

SMALL SIZE OF CASCADE GLACIERS AT MAXIMUM STAND OF THE PUGET GLACIER

The absence of fresh morainal materials of Cascade derivation along the shores of Cedar Lake proves that the local Cedar glacier did not extend to the lake at any time during the Vashon stage. In the valleys of the two forks of the Snoqualmie the Vashon lake clays are traceable well back into the interior of the range and maintain their uniformity in texture and lamination to the farthest up-valley remnants. This fact, together with the absence of interbedded tills, or evidence of ice thrusting, or ice-front deltas or kettles in the lake deposits, indicates that local Snoqualmie glaciers did not occupy the lower parts of these valleys when, at the maximum stand of the Puget Glacier at their mouths, the valleys were the sites of lacustrine sedimentation. Fresh morainal materials occur in both valleys immediately up-valley from the lake clays, but stream-cutting and aggradation in the vicinity of the heads of the former lakes obscure critical relations and make it impossible to affirm or to deny that the snouts of local glaciers may have stood in or near the heads of the Snoqualmie lakes when they were in existence.

The absence of morainal deposits of local trunk glaciers resting on lake clays in any of the valleys indicates that the local glaciers did not advance, subsequent to the lacustrine stage, into those parts of the valleys which had held lakes.

These statements refer specifically to local *trunk* glaciers. Topographic relations at the mouths of short tributary valleys heading in cirques suggest that some high-gradient tributary glaciers may have debouched into the lakes. Coarse detrital deposits, lying in the main valleys at tributary mouths and known to have been derived from the tributary basins on lithologic grounds, support this view but are not regarded as being decisive, for the writer knows of no criteria, applicable to small exposures, for distinguishing between materials laid down by ice directly and those deposited in torrential fan deltas by streams flowing from glaciers.

It appears, therefore, that local trunk glaciers were notably small at the time of maximum extent of the Puget Glacier and that they did not attain large size at any subsequent time. Relations at the heads of the former lakes in the Snoqualmie valleys, where the lake

clays are in juxtaposition to fresh morainal materials, suggest the interesting possibility that the local glaciers may have been somewhat larger during an *early phase of the last glacial stage* previous to the maximum stand of the Puget Glacier.

PROBABLE EARLY VASHON EXTENSION OF CASCADE GLACIERS

The straightness and U-form of the Snoqualmie-Cedar valleys, hanging-tributary relationships, and the existence in the Middle Fork and Cedar valleys of streamlined rock hills isolated from the valley sides by minor channels indicate the former presence of vigorous valley glaciers that may have extended to the Snoqualmie embayment or beyond. It is evident in the Cedar Valley that these features must have been formed during a pre-Vashon stage, since the parts of the valley in which they occur do not carry fresh drift. The conclusion that local Cascade glaciers were notably larger during earlier glacial stages than during the last stage is in harmony with known relations of Cascade valley glacier moraines of several ages on the east flanks of the range. It follows that general topographic relations which prove the former existence of large local glaciers in the Snoqualmie valleys cannot be used to prove that these glaciers attained large size during an early phase of the last glacial stage.

The best available evidence bearing on this problem is provided by an exposure of local till in the South Fork Valley at the present river level 3 miles down-valley from the head of the former glacial lake (Fig. 10). The till is identified as such by its sheeted structure and the presence of striated boulders and is ascribed to the South Fork trunk glacier because of a predominance of granodiorite of up-valley derivation, unlike the local valley side rock and wholly different from the rock types in the Puget moraine at the valley mouth. The till is overlain by talus which is, in turn, overlain by lake clays. In the immediate vicinity it rests on bedrock or on disturbed talus. The till is not notably clayey, and directed search has failed to discover in it the clay fragments that would be expected had the glacier been active while the lake was in existence. Stratigraphic relations of till, talus, and clay in this exposure indicate

that the local glacier had advanced at least to this point, and had withdrawn, at some time before the lake was formed.

The critical question is, of course, whether the till dates from the last glacial stage or from an earlier stage. The till is perfectly fresh and shows none of the cementation which characterizes deeply buried pre-Vashon tills elsewhere in the Puget area. There is no weathering between till and talus or between talus and clay. This is particularly

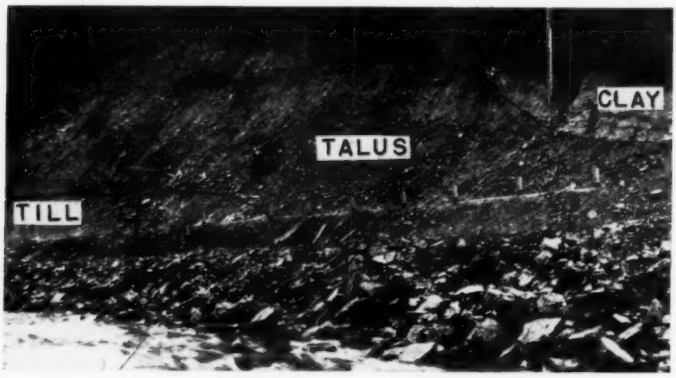


FIG. 10.—View showing relations of South Fork till, talus, and lacustrine clay. The till-talus and the talus-clay contacts slope toward the stream, and the face of the bank cuts the till-talus contact in the middle of the photograph at a low angle. The actual thickness of talus between till and clay is therefore considerably less than the apparent thickness.

significant because the three members are parts of a continuous depositional sequence with no possibility of removal of a weathered rind from any of the members in the process of emplacement of the next overlying member. The absence of the remains of even the simplest types of plants at the till-talus or at the talus-clay contacts indicates that the interval between the deposition of these members was very short, for surfaces in comparable topographic situations at the present time are quickly overgrown by vegetation.

The problem of the age of the till thus narrows down to a question as to the length of the interval represented by upbuilding of the talus. That the talus could have, and probably did, accumulate in

a short period, measurable in a few tens or hundreds of years, is suggested by the precipitous slopes from which it was shed and by the very rapid rate of talus accumulation at the base of cliffs from which valley glaciers are now withdrawing in the Cascade Range. If the period of talus accumulation was short, the underlying till is clearly Vashon in age. Nevertheless, however remote, the possibility must be recognized that the upbuilding of the talus may have occupied a full interglacial stage. The writer's conclusion that the till is Vashon is therefore tentative.

If this age assignment of the till be correct, it appears that the South Fork glacier advanced during the Vashon stage at least to the point where the till occurs and that it had retreated at least 3 miles (the distance to the up-valley end of the perfectly laminated clays) before lakeing of the valley. The talus, which also antedates the lake, was formed during this interval of retreat. Lakeing of the valley, signaled by the clays over the talus, registers the advance of the Puget Glacier to the valley mouth.

These relations suggest that the South Fork local glacier had attained considerable length during an early phase of the Vashon glacial stage and that it was in full retreat at the time when the main Puget lobe was actively advancing from its Canadian source areas. The difference in time between the attainment of maximum extent of alpine glaciers, on the one hand, and a major lobe of a continental glacier, on the other, if demonstrated for the western Cascade front as a whole, would have a direct bearing on climates and on the nourishment of the two unlike types of glaciers in this region during the Pleistocene. Closely related to this problem is the remarkably small size of the Cascade trunk valley glaciers during the maximum stand of the Vashon Puget Glacier. These are evidently regional problems, and discussion of their theoretical aspects is deferred to the report mentioned in the introduction.

It may occur to the careful reader that the fact that the Cedar valley glacier did not extend to the lower parts of the Cedar Valley during any part of the Vashon stage does not comport with the view that the South Fork glacier did occupy the lower portion of its valley during an early phase of the Vashon stage. These statements are not, however, incompatible, for the South Fork Valley drains a higher

and much more intensely glaciated part of the Cascade Range than the Cedar; and it is entirely reasonable to suppose that its trunk glacier was considerably larger than the Cedar glacier during any and all stages of glaciation.

DEGLACIATION OF THE SNOQUALMIE EMBAYMENT

DOWNMELTING FROM THE MORAINAL EMBANKMENTS

It has been shown in earlier sections that meltwaters from the Puget Glacier discharged into the Snoqualmie-Cedar lakes and escaped from them in ice-marginal channels during the upbuilding of the terminal moraines. That the drainage lines were marginal at the maximum stand of the ice and at the beginning of retreat is indicated by the fact that all three major streams began their postglacial incision at the extreme southern ends of the embankments which block their valley mouths (Figs. 1 and 5). The same relationship is seen in the minor streams draining Lakes Calligan and Hancock (*LC* and *LH*, Fig. 1), just north of the Snoqualmie embayment; it is repeated in every valley along the Puget-Cascade Mountain front, excepting only those in which impounded lakes found outlets to the south through low divides in the mountains remote from the eastern edge of the Puget lobe.

That meltwater drainage continued to occupy marginal courses during downmelting from the terminal morainal embankments is indicated by the terrace forms which characterize the ice-contact slopes. A single broad terrace which interrupts the ice-contact face of the Middle Fork embankment is shown even by the 100-foot contours of the standard topographic sheets (Fig. 1). Details are best seen in the field on the face of the Cedar embankment, which is cleared of timber; they are accurately portrayed by 10-foot contours on the map reproduced here as Figure 3 and are discussed in the explanation accompanying the map. Some of the terraces may have been produced in part by accretion of morainal detritus against the embankment faces during retreatal stands. Concave interterrace scarps meeting in cusps, and some cross sections, prove that the greater number were *cut* into the earlier deposited drift of the embankments by meltwater floods flowing between the lowering ice front and the embankment faces.

The existence of vigorous marginal drainage along the Snoqualmie-Cedar ice-contact slopes is of special interest because of its bearing on the condition of the Puget ice during the first stage of deglaciation of the Snoqualmie embayment. Marginal drainage implies a marginward slope of the ice surface and a relative absence of subglacial channels—conditions which are to be expected in active, as opposed to stagnant, ice. This deduction is confirmed by studies of Alaska glaciers before, during, and after the surge of advance between 1899 and 1909, for von Engeln and others have shown that, in that region, marginal drainage was specifically associated with vigorous forward movement and was not present during earlier and later phases of quiescence or stagnation.[10] The terraced and channeled form of the Snoqualmie-Cedar ice contacts and the complete absence of projecting crevasse fillings and other features characteristically associated with stagnant ice is here interpreted to mean that the Puget Glacier was active continuously or pulsationally during the period of downmelting from its terminal moraine.

Terracing of the Cedar ice-contact slope during retreatal stands of the Puget Glacier may have a bearing on the cause of the failure of the Cedar reservoir, for scouring action by meltwaters in the process of terrace-cutting tended to thin, and locally may have removed completely, the relatively impervious till seal on the north face of the Cedar embankment.

THE SALLAL MORAINE

A broad, hummocky morainal ridge, parallel with but approximately 500 feet lower than the embankment crests, partly encircles the eastern end of the Snoqualmie embayment (Fig. 1). It is here termed the "Sallal moraine," from a name in local usage (Fig. 11). The absence of any notable difference in weathering of the materials comprising the Sallal moraine and those of the great terminal morainal embankments indicates that there was no considerable lapse of time between the construction of these features. It is uncertain whether the Sallal moraine was built during a readvance of the Puget Glacier after a minor retreat or whether it is a recessional

[10] O. D. von Engeln, "Phenomena Associated with Glacier Drainage and Wastage," *Zeitschr. Gletscherk.*, Vol. VI (1911), pp. 104–50.

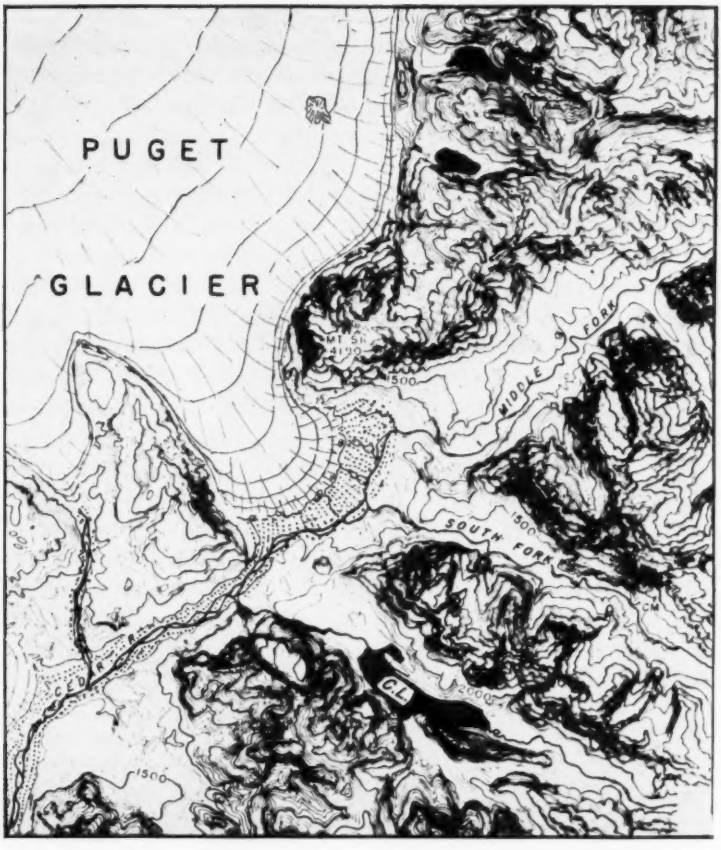


FIG. 11.—Recessional stand or readvance of the Puget Glacier (Sallal substage). Note that the Cedar spillway was re-excavated, and the Middle and the South Fork lakes were drained before or during the Sallal substage. The Middle and the South forks may have flowed westward under the margin of the Puget Glacier rather than in the marginal course shown on the map. The course of the North Fork close against the mountain front was determined by the position of the ice front during the Sallal substage (see North Fork, Fig. 12).

moraine formed by active ice during a retreatal stand of some duration. In any case, it seems desirable to recognize a "Sallal substage" during the waning of the Vashon glacial stage. In contradistinction to it the period of maximum extent of the Puget Glacier, represented by the terminal morainal embankments, may be termed the "Embankment substage."

The mouths of the valleys of the two forks of the Snoqualmie were not blocked by the Puget Glacier during the Sallal substage. It is evident, therefore, that this substage does not enter into the sequence of events described in the valley of the South Fork of the Snoqualmie, namely, early advance of the South Fork valley glacier and its retreat for some distance before lakeing of the valley as a result of advance of the Puget Glacier into the valley mouth. In other words, it appears that the greatest Vashon extension of the South Fork local glacier occurred during an early phase of the Embankment substage of the Puget Glacier and that the local South Fork glacier was retreating at the time when the Puget Glacier attained its maximum extent. The Sallal substage of the Puget Glacier may correspond with a minor readvance or retreatal stand of the local glaciers suggested by bulky moraines lying in the Snoqualmie valleys far back in the interior of the Cascades. The early Vashon advance of the South Fork glacier (early Embankment substage of Puget Glacier) and the late Vashon stand of that glacier (Sallal substage of Puget Glacier) may be synchronous with the Leavenworth and Stuart stages recognized by Ben M. Page in the Wenatchee Valley on the east flanks of the Cascade Range.[11]

FINAL STAGE OF DEGLACIATION—THE ORIGIN OF SNOQUALMIE FALLS

In the Snoqualmie-Cedar area the Cascade Mountain front is bordered by a broad piedmont plateau at a general level of 1,000 feet, contrasting markedly with the lower trough-drumloidal topography of the axial parts of the Puget lowland. The plateau appears to be a preglacial foothill belt now leveled by drift, through which protrude numerous *roches moutonnées* (Fig. 1). The Snoqualmie

[11] "Multiple Alpine Glaciation in the Leavenworth Area, Washington," *Jour. Geol.*, Vol. XLVII (1939), pp. 785–815.

River flows northwestward across this belt in a valley made up of two unlike segments separated by Snoqualmie Falls. The upper segment is the wide, flat floor of the Snoqualmie embayment, traversed by the river in looping meanders, standing 450–500 feet above sea-level or 500 feet below the level of the drift plateau. The river descends to the lower segment in a vertical drop across a ledge of early Tertiary basalt, the height of the falls being 268 feet, 100 feet higher than Niagara.

These relations at once suggest, as contended by Bretz, that the drift plateau was formerly continuous along the mountain front and that the valley was produced by postglacial incision of the Snoqualmie, the falls resulting from local superimposition of the stream on a buried rock hill.[12] Deduced consequences of this hypothesis do not, however, match observed field relations. Had the flat floor of the Snoqualmie embayment been produced by downcutting from the level of the drift plateau, it should be flanked by stream terraces cut in drift and rock-defended. The western part of the embayment flat is, in fact, bordered by glaciated rock hills still mantled in part by till; depressions among the hills show typical morainic topography or are irregular extensions of the valley floor of such form that they could not have been produced by the meanders of the Snoqualmie. It appears, therefore, that the embayment floor was produced by filling rather than downcutting in late glacial or postglacial time. The presence of over 250 feet of silt without pebbles in a well drilled in the town of Snoqualmie in the axial part of the valley flat 1 mile east of the falls supports this view and proves that the filling is lacustrine. Similar bedded clays and silts are well exposed in undercut banks on the south side of the lower Snoqualmie Valley segment west of the falls, where they terminate upward in rather extensive flats at the 500-foot level. The occurrence of lake sediments east and west of the falls is particularly significant, since it suggests that during the lacustrine stage the present separation of the valley into contrasted lower and upper segments separated at the falls did not exist (Fig. 12).

[12] *Op. cit.*, p. 224.

FIG. 12.—The origin of Snoqualmie Falls. Snoqualmie Lake (*SL*) in the lower trunk valley, was bisected by the fan-topped delta of Tokul Creek. On draining or filling of the lake, the trunk Snoqualmie River took a course around the distal edge of the delta and was superposed on a rock spur of the preglacial south valley side.

The steep scarp descending from the upper to the lower segment of the valley exposes a thick sequence of detrital deposits, largely sand and gravel, with the base less than 200 feet above sea-level. Bedrock outcrops at the falls at 500 feet and at a similar elevation in the gorge of Tokul Creek, $1\frac{1}{2}$ miles to the north, evidently mark the southern and northern limits of the preglacial Snoqualmie Valley in which these deposits lie. The structure of the fill, here exposed from valley side to valley side, is everywhere deltaic; and radial dips in the foresets, as shown on Figure 1, identify Tokul Creek as the delta-building stream. The fan form of the surface of the delta, where not modified by later swinging of the Snoqualmie, slopes southward completely across the filled valley to the lip of the falls.

These relations, taken together, indicate that during the last stage of deglaciation of the Snoqualmie-Cedar area a tongue of ice lay in the deep preglacial valley of the Snoqualmie River. The irregular trend of the valley and the absence of associated recessional moraines suggest that it was stagnant. This view is confirmed by the fact that similar valleys in adjacent areas, which, by chance, escaped postglacial modification by through-flowing streams, retain all the distinctive features of ice stagnation. Melting of the ice tongue from the part of the Snoqualmie Valley here under consideration permitted the formation of a lake, with its water-level at 450–500 feet. The lake was filled largely by bedded silts and clays, but the late glacial Tokul Creek built a sand-gravel delta which bisected it, producing a situation somewhat similar to that at Interlaken in Switzerland. When, finally, incision of the Snoqualmie into the lacustrine fill began, the stream assumed a course around the distal end of the fan-topped delta and was thus superposed on a rock spur of the preglacial south valley side. The river has since been fixed in position at this point; up-valley and down-valley it has swung widely, slicing away and terracing the Tokul deltaic deposits. At one stage in this process it narrowly escaped intercision with Tokul Creek, which would have withdrawn the river from the falls and permitted it to cut a deep trench in the flat floor of the Snoqualmie embayment up-valley.

It should be noted that this explanation places the falls of the trunk Snoqualmie in a wholly different category from the falls and

gorges of the South Fork and the Cedar River. The latter streams were displaced from their preglacial valleys by ice directly and are associated with terminal moraines, while the trunk Snoqualmie was displaced by a late-glacial fan delta built by a tributary into a lake after complete withdrawal of the ice from the immediate vicinity. All cases of superposition and gorge-cutting of trunk streams along the Cascade-Puget Mountain front fall into either one of these two classes.

ACKNOWLEDGMENTS.—It is a pleasure to acknowledge indebtedness to Professors Douglas Johnson, J. D. Barksdale, H. A. Coombs, and R. F. Flint for discussion of problems of the Snoqualmie-Cedar area and/or critical reading of the manuscript. Responsibility for interpretations rests, of course, wholly on the writer.

THE EFFECTS OF ABRASION ON THE SIZE, SHAPE AND ROUNDNESS OF ROCK FRAGMENTS

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ABSTRACT

Earlier work on the abrasion of rock fragments in tumbling barrels is reviewed, and the main principles are summarized. Most earlier work concerned one or another aspect of abrasion without including interrelations among particle characteristics during wear. Experimental data are presented on size, sphericity, and roundness during abrasion, and the relative rates of size reduction, rounding, and shape change are determined. It is found that the coefficient of rounding is largest, the coefficient of shape change is intermediate, and the coefficient of size reduction is smallest. An analytical theory of abrasion is developed on the basis of the experimental data, and it is found that simple exponential functions yield at least first approximations to the size and shape aspects of particle wear. Rounding is a more complex process, controlled in part by the shapes of the particles.

Geological applications of tumbling-barrel data are discussed, and it is emphasized that wear of rock fragments is only one aspect of the changes which sediments undergo during transportation. The selective effects of transportation, in terms of size, shape, and density, must be taken into account in any analysis of actual field relations.

INTRODUCTION

Numerous experiments have been conducted on the artificial abrasion of rock fragments, and the geological literature on tumbling barrels is moderately extensive. In many instances some particular aspect of particle wear was studied, but the writer is not aware of any general theory of the tumbling barrel in terms of the interrelations among size, shape, roundness, lithology, and rigor.

The geological interpretation and application of abrasion studies are complicated by the presence of two very distinct effects in the natural transportation of sediments. The first is the actual wear and breakage of the particles en route, and the second is selective transportation. It does not necessarily follow, for example, that an observed decrease in average particle size along a stream is wholly due to abrasion, because it is possible that the smaller particles may have outstripped larger ones during transport. It cannot readily be determined from the observed data alone how much of the apparent size decrease is due to one or the other of these two effects.

Abrasion and selective transportation may be separated in the

laboratory, and each one studied independently by experiment. A load subjected to wear in a tumbling barrel is free from selective transportation effects because the load retains its identity during the experiment. The tumbling barrel may thus be used in one aspect of the problem. The data of tumbling-barrel experiments may be supplemented by flume experiments in which the selective effects of transportation alone are considered. The relatively short distance of transport in flumes eliminates appreciable wear, so that selective transportation itself may be directly observed, in terms of its effect on size, shape, and other sedimentary characteristics under varying conditions. Combined data from tumbling barrel and flume should furnish a more satisfactory basis for the evaluation of field conditions of sedimentation, as well as lay the foundation for the reconstruction of ancient sedimentary environments.

The present paper indicates some of the interrelations which exist among size, shape, and roundness during abrasion. The work is confined to particles in the pebble and cobble range under conditions of abrasion alone. It is understood that the generalizations advanced may not apply to smaller particles. A similar study of sand would probably demonstrate the influence of some factors which are negligible among larger particles, as well as eliminate other factors which may apply only to particles above some critical size limit. The location and study of transition zones between fields of operation of coarse-sediment laws and fine-sediment laws constitutes in itself a major field of investigation.

REVIEW OF TUMBLING-BARREL STUDIES

A number of papers which discuss tumbling-barrel data are confined to small particles. For the present paper, studies of coarse particles are more pertinent. The earliest application of the tumbling barrel to geological problems was made by A. Daubrée.[1] His apparatus consisted of a horizontal axis and frame, to which four iron or stone jars were fastened. During operation the jars were given a combined rotation and revolution. In most of his experiments Daubrée's barrel had a rotation equivalent to about 2 miles per hour.

[1] *Etudes synthétiques de géologie experimentale* (2 vols.; Paris: Dunod, 1879).

Among the principles which Daubrée discovered, the following are most important for the present study:

1. Angular particles in the pebble range ("fist-size to nut-size") become rounded very rapidly; even granite and quartz particles are thoroughly rounded after movement equivalent to 25 km.
2. The rate of wear is noticeably more rapid for angular than for well-rounded particles.
3. The rate of wear is more rapid for large than for small particles.
4. The rate of wear is greater for nonresistant than for resistant particles (i.e., feldspar wears more rapidly than flint).
5. The rate of wear is a function of the rigor of the process. (This principle for large particles is implied rather than explicitly stated.)
6. Wear may be due either to abrasion or to breakage; the effects of the two are not the same.
7. The product of abrasion is almost entirely mud rather than sand.
8. The apparent decrease in size of particles along natural streams is not due entirely to wear; selective transportation is also involved.

The next comprehensive study of abrasion, as far as the writer is aware, was made by C. K. Wentworth in a series of papers[2] dating from 1919 to 1931. In his first paper Wentworth pointed out that the fundamental characteristics of sedimentary particles are their size, shape, surface texture, and lithologic composition. He emphasized that a number of factors control the changes which occur in size, shape, and surface texture during abrasion, although it is not explicitly stated that each factor applies to all three characteristics:

Size Shape Surface texture	Controlled in part by	Size of particle Angularity Kind of rock Kind of motion Violence of motion Distance Size of associated particles Number of associated particles

Wentworth's realization of these interrelations led to the important conclusion that each relation could be studied independently by

[2] "A Laboratory and Field Study of Cobble Abrasion," *Jour. Geol.*, Vol. XXVII (1919), pp. 507–22; "The Shapes of Pebbles," *U.S. Geol. Surv. Bull. 730* (1922), pp. 91–102; "A Field Study of the Shapes of River Pebbles," *U.S. Geol. Surv. Bull. 730* (1922), pp. 103–14; "Pebble Wear on the Jarvis Island Beach," *Washington Univ. Studies Sci. and Tech. No. 5* (new ser., 1931), pp. 11–37.

experiment. Wentworth himself studied several. His early experiments were conducted in a metal drum lined with soft wood. This apparatus was subsequently modified several times, and the latest variation[3] was a much improved gear-driven abrasion mill with a water feed which circulated clean water through the drums.

Several of Wentworth's numerous contributions to the theory of abrasion are of particular interest here:

1. The fundamental characteristics of sediments which change during abrasion are size, shape, and surface texture.
2. Factors which control the changes in sedimentary characteristics during abrasion include, among others: size, angularity (roundness), kind of rock, violence (rigor) of the abrasive process, and distance.
3. Roundness as a function of distance transported is represented by a graph which rises rapidly at first and then more gradually. In exceptional instances roundness may even decrease after reaching a maximum value.
4. Size as a function of distance is a negative exponential function. (This is implied in Wentworth's 1931 graphs, although the relation is not expressly stated. In his 1919 paper the relation was shown as a nearly linear function.)
5. Roundness may be a function of size as well as of hardness. In Wentworth's Russell Fork paper (1922) the relation suggests a power function.

The third important contributor to tumbling-barrel theory is P. Marshall, who published a series of papers on the subject[4] from 1927 to 1929. Marshall confined his studies to natural beach sediments which were subjected to abrasion in a Deval machine with iron cylinders inclined 30° to the horizontal axis. During each revolution the particles fell several inches and slid the length of the cylinder. The particles moved with a speed equivalent to about 1 mile per hour.

Marshall found that a load of 5 kg. of natural beach gravel, abraded for 360 hours, lost 21.25 per cent of its weight and yet displayed no noticeable change in sorting.

Further study of Marshall's original data, in terms of the statistical parameters of the size distribution, verifies the negligible change in the sorting. The log standard deviation, σ_ϕ, was found to be 0.95 at the start of the experiment, and 0.87 after 360 hours. Physically this means that initially there were 1.90

[3] "Pebble Wear on the Jarvis Island Beach," *op. cit.*

[4] "The Wearing of Beach Gravels," *Trans. New Zealand Inst.*, Vol. LVIII (1927), pp. 507–32; "Colloid Substances Formed by Abrasion," *ibid.*, Vol. LIX (1928), pp. 609–13; "Beach Gravels and Sands," *ibid.*, Vol. LX (1929), pp. 324–65.

Wentworth grades in the central part of the size distribution, whereas at the end there were 1.74 Wentworth grades in the same central portion. Thus the sorting improved only very slightly during the experiment. Moreover, the geometric mean diameter proved to be 15.5 mm. both at the beginning and end of the experiment, so that no apparent change occurred in the mean size of the distribution. Finally, the skewness of the distribution was essentially negligible during the experiment. Marshall's work thus showed that under some conditions of abrasion there may be no significant changes in any of the three main size parameters of the sediment.

A similar analysis applied to intermediate stages of the experiment showed essentially the same parameters throughout, with minor non-systematic fluctuations. Marshall himself stresses the stability of the proportions in the several size grades, and it seems reasonable that his interpretation of the development of equilibrium proportions under conditions of abrasion is sound. The use of natural beach gravel probably means that the equilibrium stage had been reached by the time the sample was taken, and the movement of the machine was not sufficiently unlike natural movement to destroy the effect.

In addition to his findings for beach gravel, Marshall sharpened the terminology of particle wear by defining the terms "abrasion," "impact," and "grinding." "Abrasion" is defined as the mere rubbing of pebble against pebble; "impact" is the effect of definite blows of relatively large pebbles on others, relatively small; and "grinding" is the crushing of small grains by the continued contact and pressure of pebbles of relatively large size. Wentworth discussed these terms[5] and suggested that the general term "wear" be used to include all three. In the present paper these definitions are accepted, although the writer agrees with Wentworth that no restrictions should be placed on the relative sizes of the pebbles involved in impact. Impact may, in fact, be caused on particles by others of smaller size.

Among other contributions by Marshall was a substantiation of Daubrée's earlier findings that mud, rather than sand, is the main product of abrasion. A summary of several of Marshall's findings follows:

1. The wear of particles may be divided into three separate processes: abrasion, impact, and grinding.
2. During abrasion a mixture of various-sized particles tends to approach equilibrium proportions among the several sizes present.

[5] "Pebble Wear on the Jarvis Island Beach," *op. cit.*

3. After equilibrium is approached, prolonged abrasion on a frequency distribution of particles may not significantly change the parameters of mean size, degree of spread, or skewness.
4. If a sample of gravel contains an appreciable amount of fine material (under about 4 mm. diameter), the smaller particles are crushed to silt and finer material, and the mean size of the remaining gravel may actually increase with abrasion.

Within recent years A. Schoklitsch[6] published a fairly exhaustive study of pebble abrasion in terms both of artificial wear in tumbling barrels and natural wear along streams. As early as 1914 he[7] had made experiments which substantiated Sternberg's law of size reduction, and in the more recent paper this work was extended to other aspects of abrasion theory. Sternberg's law is important in the present development, and it may be briefly described here. Sternberg[8] showed in 1875 that the weight of a pebble decreases exponentially during abrasion, in accordance with a function of the type

$$\frac{w}{w_0} = e^{-as},$$

where w is the weight at any distance s, w_0 is the initial weight of the pebble, e is the base of natural logarithms, and a is a coefficient of size reduction, a constant under given experimental conditions.

In his earlier work Schoklitsch used a stationary tub in which a stream of water entered from the side and swirled the pebbles around. A smaller cylinder in the center, concentric with the outer walls, was used for overflow. In his later study Schoklitsch used a tumbling barrel 0.7 meter in diameter, which rotated on a central shaft. The pebble or fragment to be observed was placed in the barrel with about 2 liters of finer gravel and enough water to cover the load. Observations were made on the single test pebble, but during wear it was part of the general load.

The first use of the barrel was to resubstantiate Sternberg's law

[6] Über die Verkleinerung der Geschiebe in Flussläufen," *Sitzungersber. d. Akad. d. Wissensch. in Wien*, Vol. CXLII, Part IIa (1933), pp. 343–66.

[7] *Über Schleppkraft und Geschiebebewegung* (Leipzig, 1914), p. 12.

[8] Reference cited by Schoklitsch as *Zeitschr. f. Bauwesen*, Vol. XXIX (1875), p. 483.

in refutation of work by F. Düll,[9] who sought to show that the abrasion law was a power function. Schoklitsch's detailed experimental data leave no doubt of the essential validity of Sternberg's law. Among the numerous factors studied by Schoklitsch, the following are of principal interest here:

1. Sternberg's law is valid during abrasion and may be expressed not only in terms of weight but also approximately in terms of volume or length for given pebbles.
2. The coefficient a of Sternberg's law is a constant for a particular kind of rock under given conditions, but it varies as the fourth root of the velocity of movement of the tumbling barrel.
3. The abrasion of a particle is controlled in part by the size distribution of the material with which it is associated. An increase in the mean size of the associated material causes a linear increase in the coefficient a.
4. Wear by impact (breakage) is more rapid than wear by abrasion; in some instances more than ten times as great.

RELATIONS AMONG SIZE, SHAPE, AND ROUNDNESS DURING ABRASION

The literature reviewed above contains abundant experimental data on certain aspects of abrasion, but none seems to be available on the interrelations among size, shape, and roundness during abrasion. To investigate these subjects a load of rock fragments was abraded for varying intervals of time, and after each interval measurements were made of the size, sphericity, and roundness of each of the particles.

The tumbling barrel used in the experiment is illustrated in Figure 1. It consists of a metal oil drum, 18 inches in diameter and 21 inches long. The drum is lined with oak flooring, including the ends, which have circular openings offering access to the interior during operation. The inside diameter of the barrel is 16 inches. One end of the drum is removable; during operation it is held in place with a metal clamping ring. The barrel has two reinforcing flanges which are used for driving and guiding it. The barrel is mounted on two horizontal shafts with wheels which fit the flanges. One of the shafts drives the barrel, and the other is an idler. The power is transmitted through a countershaft which permits four speeds by means of step-cone pulleys. The pulleys are connected by rubberized V-belts. The motor used is $\frac{1}{4}$ horsepower, 110 volt A.C., operating at 1,750 r.p.m. This speed is reduced so that the barrel operates at 21 r.p.m. at

[9] "Das Gesetz des Geschiebeabriebes," *Mitt. a. d. Geb. d. Wasserbaues u. d. Baugrundforschung*, Vol. I (Berlin, 1930).

its lowest speed. This speed was used in the experiment to be described; it corresponds to a motion of about 1 mile per hour. Driving torque was greatly improved by placing rubber strips over the driving wheels, and loads of 5 kg., including several liters of water, were handled effectively by this drum. The entire apparatus is mounted on a rigid framework of 2×6 timber. A revolution-counter is attached to the shaft as a check on the operation of the drum.

The load used in the experiment consisted of a selected set of twenty-seven limestone fragments obtained from a commercial

FIG. 1.—Tumbling barrel used in abrasion experiment

crusher, which assured a high angularity in the fragments. The fragments were selected for size, and within the size class a limited range of sphericities was chosen. The selection by size was by sieving within one $\sqrt[4]{2}$ interval, from 45 to 54 mm. diameter. (This is equivalent to one $\phi/4$ interval, from -5.75 to -5.50.)[10] The average sphericity of the fragments was 0.65, the average roundness was 0.13, and the arithmetic mean pebble weight was 155 gm. The appearance of the original set of pebbles is given in Figure 2, upper left, photographed before the experiment was begun.

The twenty-seven fragments were placed in the tumbling barrel

[10] W. C. Krumbein and F. J. Pettijohn, *Manual of Sedimentary Petrography* (New York: D. Appleton-Century Co., 1938), chap. iv.

with sufficient water to keep the load covered during the operation, and the barrel was run for 10 minutes. The fragments were then removed, washed, dried, and all measurements were repeated. The

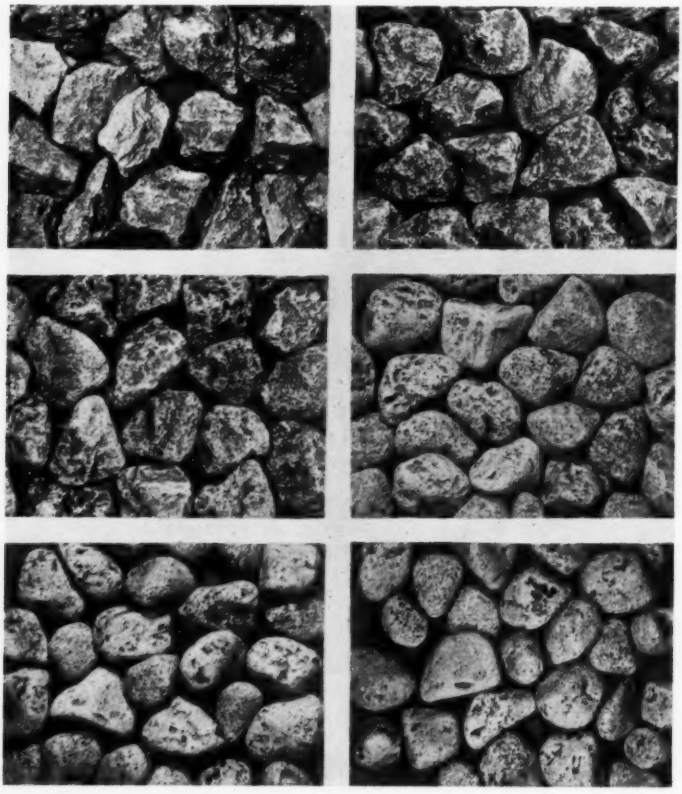

FIG. 2.—Limestone fragments at various stages of abrasion. *Upper left*, original fragments; *upper right*, 0.5 mile; *left center*, 1.0 mile; *right center*, 5 miles; *lower left*, 9 miles; *lower right*, 20 miles.

experiment was continued with intervals of varying magnitude as the experiment progressed; in all, a total running time of 20 hours was used, which carried the experiment beyond the point where the

arithmetic mean weight of the pebbles was reduced to half its original value. Fourteen sets of data were obtained.

During the experiment the fine material which resulted from the abrasion was collected and examined. Daubrée's and Marshall's observations of the production of mud were amply substantiated, but in the first two runs a number of limestone chips were also found. These were interpreted as impact breakage due to weaknesses from crushing in the original pebbles which could not be detected by eye.

TABLE 1

EXPERIMENTAL VALUES ON LIMESTONE FRAGMENTS

Time in Minutes	Distance in Miles	Arithmetic Mean Weight in Grams	Arithmetic Mean Roundness	Arithmetic Mean Sphericity
0	0.00	155	0.13	0.65
10	0.17	150	.26	.66
20	0.33	148	.38	.67
30	0.50	146	.44	.67
60	1.00	140	.50	.68
120	2.00	131	.55	.69
180	3.00	124	.56	.70
240	4.00	118	.58	.71
300	5.00	113	.59	.72
420	7.00	106	.60	.73
540	9.00	98	.61	.74
720	12.00	85	.61	.75
960	16.00	73	.63	.76
1,200	20.00	60	0.64	0.77

With the exception of these initial effects, it appeared that abrasion alone was operative during the main part of the experiment.

The qualitative results of the experiment are shown in Figure 2, which illustrates the load after various distances of transport. These photographs show that rounding occurs rapidly, and at the end of the 20 hours the fragments have the appearance of well-rounded pebbles. The relative sizes of the pebbles during the experiment are correctly shown in the photographs, which are all to the same scale.

Table 1 lists the experimental data. The arithmetic mean particle weight was computed by dividing the total dry weight of the particles after each run by their number. The roundness was obtained by measuring the fragments after each run according to the method

of Wadell[11] and computing the arithmetic mean. The sphericity of each particle was measured by the standard-volume method[12] and the arithmetic mean computed. The choice of parameters used in presenting the data is one of convenience.

Figure 3 shows the results of the experiment graphically. The curve of arithmetic mean weight (size) descends rapidly from its initial value of 155 gm., but the curve shows a continuously decreasing slope as distance increases. The graph of roundness rises rapidly

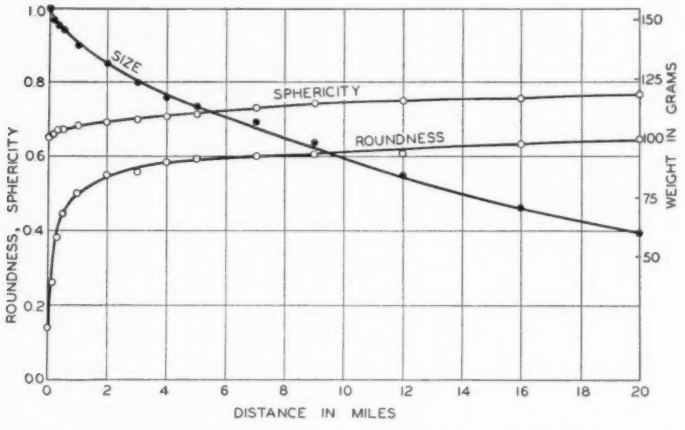

FIG. 3.—Size (weight), roundness, and sphericity of limestone fragments as functions of distance during abrasion.

from its initial value but flattens noticeably beyond the first 3 or 4 miles; toward the end of the experiment there is practically no further increase in rounding. Sphericity shows only a slight change during the entire experiment. From an initial value of 0.65 the graph rises moderately rapidly during the early part of the experiment, but during the later stages there are no marked changes in the numerical values.

The experimental data present several suggestions regarding the nature of the changes which take place during abrasion. It seems fairly clear that size will be continuously reduced until ultimately

[11] *Ibid.*, chap. xi. [12] *Ibid.*

the particles will be worn away. On the other hand, although the roundness and sphericity increase continuously during the experiment, there is a strong suggestion that they approach limiting values which may depend upon certain initial values. That is, a pebble with noticeably different values of its three size axes may never reach the value 1.00 for either its roundness or sphericity. Mathematically this may be expressed by saying that the roundness and sphericity approach asymptotes which depend in part upon initial size or shape or both. In some instances the value of the asymptotes may be 1.00 for both roundness and sphericity; in other instances neither may reach this value, nor need the two asymptotes have the same value. Within the range of the present experiment roundness appears to approach an asymptote slightly higher than 0.64, and sphericity approaches an asymptote slightly above 0.77. In contrast to both the roundness and sphericity curves, which flatten very appreciably during the later stages of the experiment, the size curve continues to descend in a broad sweep. Thus, zero is the most likely asymptote approached by size.[13]

When experimental data yield curves which seem to approach asymptotic values, a convenient manner of comparing the rates of change of the several variables is by means of so-called "half-values." For example, the initial arithmetic mean weight of the fragments is 155 gm. On the assumption that the asymptote of the size curve is zero, the half-value is 77.5 gm. This value is reached in approximately 14 miles, which defines the "half-distance," or that distance over which the original value is reduced by half. In similar manner, the initial roundness is 0.13, and the final observed value is 0.64. As a first approximation assume that the final value is near the asymptote. Then the half-value is the initial value plus half the difference between the first and last values: $0.13 + (0.64 - 0.13)/2 = 0.385$. The roundness curve reaches the value 0.385 in approximately 0.4 mile, which may be taken as the half-distance of the

[13] The asymptotes approached by the curves in any experiment depend in part upon the total distance covered. In other experiments by the writer, extending over varying distances of abrasion, the roundness and sphericity curves continue to rise very slowly, in marked contrast to their abrupt rise in the early stages of the experiment. The size curve shows no corresponding flattening.

roundness curve. Finally, on the assumption that sphericity approaches an asymptote near the final value of 0.77, the half-value is $0.65 + (0.77 - 0.65)/2 = 0.71$. This value is reached by the sphericity curve at a distance of approximately 4 miles. The reciprocals of the half-distances may be used directly as coefficients. Thus, $k_1 = 1/14 = 0.071$ may be interpreted as the size coefficient, $k_2 = 1/0.4 = 2.5$ may be interpreted as the rounding coefficient, and $k_3 = 1/4 = 0.25$ may be taken as the coefficient of shape change.

In order to compare the rates at which size, sphericity, and rounding occur, one may express the coefficients as ratios. Then the ratio k_2/k_1 expresses how much more rapidly roundness reaches its half-value than size reaches its own half-value. The resulting number may be interpreted physically as the ratio of the rate of rounding to the rate of size decrease. In the present instance $k_2/k_1 = 2.5/0.071 = 35$, which indicates that in this experiment rounding proceeds 35 times as fast as size decreases. Similarly, the ratio $k_3/k_1 = 0.25/0.071 = 3.5$, indicates that sphericity change proceeds 3.5 times as fast as size reduction. Finally, the ratio $k_2/k_3 = 2.5/0.25 = 10$ means that rounding proceeds 10 times as rapidly as sphericity change.

An apparent difficulty in applying these ratios to the entire experiment can be brought out by a further comparison. At the end of the 0.4 mile which represents the half-distance of rounding, the arithmetic mean weight of the pebbles has dropped only 6 per cent of its original value. This means that an appreciable increase in rounding may occur with an essentially negligible decrease in size. After the roundness curve loses its steepness, however, the apparent relations between size and rounding change considerably. That is, at the point where the roundness curve begins to flatten appreciably (between 2 and 3 miles), the weight has dropped only between 15 and 20 per cent. Thereafter the roundness increases only relatively slightly to 0.64, whereas the weight drops to about 40 per cent of the original weight. Thus, after the first rapid increase of rounding, size reduction may apparently take place relatively more rapidly than roundness increase. Actually, however, it may be shown that the several ratios remain constant during the experiment. Relations of

this kind between two or more variables are frequently encountered in certain kinds of exponential functions.

Although one cannot safely generalize from the limited data at hand, the experimental findings agree with what one may theoretically expect during abrasion, on the basis of qualitative reasoning alone. An angular pebble subjected to abrasion will increase noticeably in roundness when the sharpest corners are only slightly worn, whereas the effects of removing such corners from the pebble will not affect either the size or the shape to any significant extent. Thus it seems reasonable to expect that a fragment may become moderately well rounded without any appreciable effect on size or shape.

The concept of shape distinct from roundness is rather new, and many earlier descriptions of roundness were actually descriptions of shape, and vice versa. The stability of shape of stream and beach pebbles has been observed by numerous workers, who refer to the general triaxial form of the one and the disklike form of the other. On the whole, therefore, it seems likely that the shape may change only relatively slightly during abrasion, and that the end product will, in general, show traces of the original inherited form.

In special instances an opposite effect may occur. A homogeneous sphere with roundness and sphericity each equal to 1.0 will continuously decrease in size with no change of shape or roundness. Likewise a cube with an initial roundness of zero and a moderate sphericity will approach a true sphere in shape and roundness as abrasion proceeds and thus lose both its original form and angularity. Natural rock fragments are generally triaxial in shape, however, so that the combined effects of abrasion and selective transport tend to yield characteristic pebbles in many instances.

A cube furnishes an excellent example to illustrate the relative effects of rounding and size change during early and later stages of abrasion. The cube may be just too large to pass through a given sieve. That cube may be rounded almost to a sphere without falling through the sieve. In terms of usual methods of measuring size, therefore, there may be no apparent change in sieve data over a considerable range of rounding. In fact, in order to reduce a cube to a sphere small enough to fall through a sieve interval of one Went-

worth grade, the weight must be reduced by approximately 90 per cent. This is apparent when one recalls that the ratio of the volumes of two spheres with radii 1 and $\frac{1}{2}$ is 8:1. Hence, seven-eighths of the original sphere must be removed to halve the radius. After rounding has essentially reached its limiting value under given conditions, it is reasonable to assume that, whereas size will continually decrease, the roundness will remain approximately constant.

Wentworth's original experiment on rounding[14] was conducted with cubes, and although he does not furnish data on size and sphericity, his results may be used for a qualitative comparison with the writer's data. Wentworth's definition of roundness is not the same as that used here, because he measured the ratio of the sharpest corner on a pebble to an intercept, whereas the method of Wadell involves the ratio of the average curvature of the edges to a mean intercept. Nevertheless, the two concepts are similar geometrically and may be compared. In Figure 26 of Wentworth's paper the roundness history of four marble cubes is shown. The particles increase in roundness from 0 to 0.862 in 209 miles. The general curve resembles that of Figure 3 of this paper, except that the particles decrease in roundness after the 209-mile point.[15] The half-value of rounding is 0.431, which is reached in approximately 22 miles. Wentworth includes photographs of his cubes during abrasion, and these permit an estimate of the half-distance for size. The volume of a sphere circumscribed by a cube is 0.525 of the volume of the cube, so that the weight of the sphere is slightly more than half that of the cube. An inspection of Wentworth's photographs shows that at 94 miles the cubes have become roughly spherical, with a diameter approximately equal to an edge of the cube. As a first approximation, 90 miles may be taken as the half-distance for size. This yields the coefficient $k_1 = 1/90 = 0.011$ for size and $k_2 = 1/22 = 0.045$ for rounding. Hence the ratio $k_2/k_1 = 0.045/0.011 = 4.1$, indicating that rounding proceeded more rapidly than size reduction. The marked difference in the numerical value of this and the writer's ratio lies partly in the different methods of expressing roundness.

Wentworth's photographs also furnish some clue to the change in sphericity during abrasion. The calculated sphericity of a cube is approximately 0.7· inspection of the photographs shows that the cubes do not become true spheres during the experiment but reach a value very close to 1, say 0.95. The half-value between 0.70 and 0.95 is 0.825. From the photographs of the cubes at 31 and 54 miles, it is found that the projection sphericities of the cubes are roughly 0.8 and 0.9, so that again as a first approximation, the half-distance of sphericity may be taken at 40 miles. Hence the coefficient of shape change is $k_3 = 1/40 = 0.025$. The various ratios accordingly are: $k_2/k_1 = 4.1$ is the size-roundness

[14] "A Laboratory and Field Study . . . . ," *op. cit.*

[15] The point is discussed briefly in the appendix at the end of this paper.

ratio, $k_3/k_1 = 0.025/0.011 = 2.7$ is the size-sphericity ratio, and $k_2/k_3 = 0.045/0.025 = 1.8$ is the roundness-sphericity ratio. These ratios suggest that the rate of rounding is about four times as fast as size reduction, and about twice as great as shape change. Shape change, in turn, is about twice as rapid as size reduction. The relative order of these rates of change agrees with the writer's experiment.

Although the exact numerical values of tumbling-barrel data cannot be applied directly to conditions in nature, the general order in which the several ratios occur should be approximately the same. Wentworth has discussed the general similarity of artificial abrasion studies to natural phenomena and has shown that the general nature of the rounding curves found agrees well with natural instances.[16] The writer has also observed such instances. The roundness curve of stream pebbles in San Gabriel Canyon, California,[17] agrees in general with Wentworth's findings, and in a study of beach pebbles[18] it was found that, over a limited distance at least, the size decrease of the pebbles was of the type indicated by the present experimental data. Schoklitsch[19] also has found natural size changes along streams in agreement with experimental curves.

ANALYTICAL THEORY OF ABRASION

A complete theory of particle wear involving impact, grinding, and abrasion as they affect all possible sizes and kinds of particles is at present essentially impossible because of lack of sufficient knowledge of the physical processes involved and because of the limitations of mathematical analysis when a large number of variables is present. It is possible, however, to carry the analysis beyond present experimental bounds. This is particularly true for simple situations involving uniform conditions of rigor in the wearing process and for particles of identical size, shape, roundness, and lithology. Moreover, by confining attention to abrasion alone complexities introduced by other wearing phenomena are eliminated.

[16] "A Field Study . . . . ," *op. cit.*

[17] W. C. Krumbein, "Flood Gravel of San Gabriel Canyon, California," *Bull. Geol. Soc. Amer.*, Vol. LI (1940), pp. 639–76.

[18] W. C. Krumbein, "Sediments and Exponential Curves," *Jour. Geol.*, Vol. XLV (1937), pp. 577–601.

[19] "Über die Verkleinerung . . . . ," *op. cit.*

In developing an elementary theory to account for the observed experimental data, idealized versions of the data may be used. Figure 4 (*top*) shows the curve of size, r, descending steeply at first and then more gradually as distance s increases. An ordinate has been erected at s, rising to size r. Inasmuch as the curve starts at some initial size r_0, and the curve decreases in slope as r becomes smaller, one may make the assumption that the rate of change of size with respect to distance, dr/ds, is proportional to the size r of the particle at any point. This is essentially the approach used by Sternberg in 1875. It yields the differential equation

$$\frac{dr}{ds} = -a_1 r , \qquad (1)$$

where a_1 is a constant of proportionality and the negative sign indicates that the size decreases.

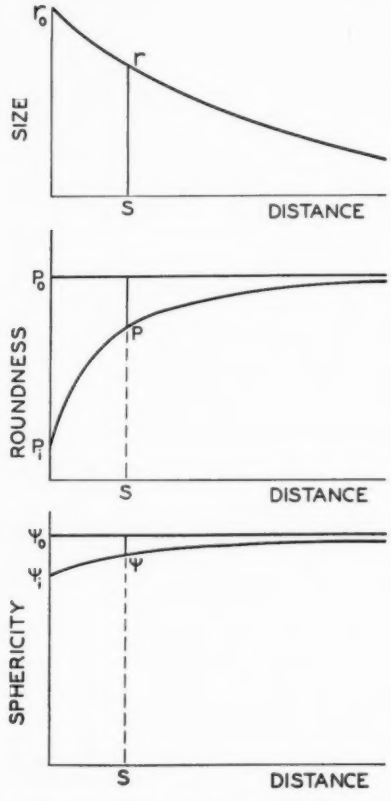

FIG. 4.—Generalized abrasion curves. *Top*, size curve; *center*, roundness curve; *bottom*, sphericity curve. The several symbols are defined in the text.

In an earlier paper[20] the writer developed an elementary theory of rounding, based on the observation that sharp edges and corners on pebbles are abraded more readily than blunt edges. This suggests that rounding is more rapid when

[20] "Flood Gravel of San Gabriel . . . . ," *op. cit.*

the particle is angular and slows down as the limiting value of roundness is approached. Mathematically one may assume that the rate of change of rounding with respect to distance, dP/ds, depends upon how far removed the roundness is at any instant from its final limiting value. This concept is illustrated in Figure 4 (*center*) where the curve of roundness, P, rises from its initial value P_i, rapidly at first and then more slowly toward its asymptote P_o. At point s an ordinate has been drawn, the solid portion of which represents the difference between the asymptote and the curve at that point; that is, the length of the solid line is $(P_o - P)$. By setting the rate of change of rounding proportional to this difference, the following differential equation is obtained:

$$\frac{dP}{ds} = a_2(P_o - P) , \tag{2}$$

where a_2 is a constant of proportionality.

The observation that the sphericity curve of Figure 3 also approaches an asymptote is indicated in Figure 4 (*bottom*). The sphericity starts at some finite value Ψ_i and approaches the asymptote Ψ_o as distance increases. As with roundness, one may assume that the rate of change of sphericity with respect to distance, $d\Psi/ds$, is proportional to the difference between the curve and the asymptote at any point. This difference is $(\Psi_o - \Psi)$, as shown by the solid part of the ordinate at s in Figure 4 (*bottom*). It is also reasonable to suppose that the sphericity change will be greater with particles of low sphericity than with particles of high sphericity. This reasoning yields a differential equation as follows:

$$\frac{d\Psi}{ds} = a_3(\Psi_o - \Psi) , \tag{3}$$

where a_3 is a constant of proportionality.

Equations (1)–(3) are standard forms which yield readily to integration. Details will be omitted here, because the solutions can be found in almost any reference book on differential equations. One intermediate step is given for later reference. The respective solu-

tions of the three equations assume the intermediate forms shown below:

$$\frac{r}{r_o} = e^{-a_1 s}, \tag{4}$$

$$\frac{(P_o - P)}{(P_o - P_i)} = e^{-a_2 s}, \tag{5}$$

$$\frac{(\Psi_o - \Psi)}{(\Psi_o - \Psi_i)} = e^{-a_3 s}, \tag{6}$$

with the following boundary conditions: when $s = 0$, $r = r_o$, $P = P_i$, and $\Psi = \Psi_i$. In the form given in equations (4)–(6), the three variables behave as negative exponential functions. These expressions may be solved for the dependent variables directly, yielding the following standard forms:

$$r = r_o e^{-a_1 s}, \tag{7}$$

$$P = P_o(1 - e^{-a_2 s}) + P_i e^{-a_2 s}, \tag{8}$$

$$\Psi = \Psi_o(1 - e^{-a_3 s}) + \Psi_i e^{-a_3 s}. \tag{9}$$

In the writer's earlier discussion of a theory of rounding,[21] the initial roundness was assumed to be 0. In the general case this may not be so, and equations (5) and (8) were developed with this in mind. The resulting expressions are accordingly somewhat more complicated than the earlier form.

The graphs of equations (7) and (9) are shown in Figure 4. The constant a_1 determines how rapidly the size curve drops; it may be called the "coefficient of size reduction"; and its value depends, as Schoklitisch has shown, on the nature of the rock, the rigor of the process, and on other factors. The constant a_2 may be called the "coefficient of rounding"; its value may depend not only on rock type and rigor but also on size and shape. Similarly, a_3 may be called the "coefficient of shape change"; its value will depend upon various factors also. For any given kind of rock, under given conditions, the several a's should be constants, but it need not be true that $a_1 = a_2 = a_3$.

[21] *Ibid.*

The general agreement in appearance of the curves in Figure 4 with the experimental data is not sufficient to warrant the conclusion that the theory is acceptable. Fortunately, there are relatively simple methods of testing the theory by means of equations (4)–(6), which represent transformed statements of the final equations shown in Figure 4. If the quantities on the left-hand side of equations (4)–(6) are plotted on a logarithmic scale against distance on an arith-

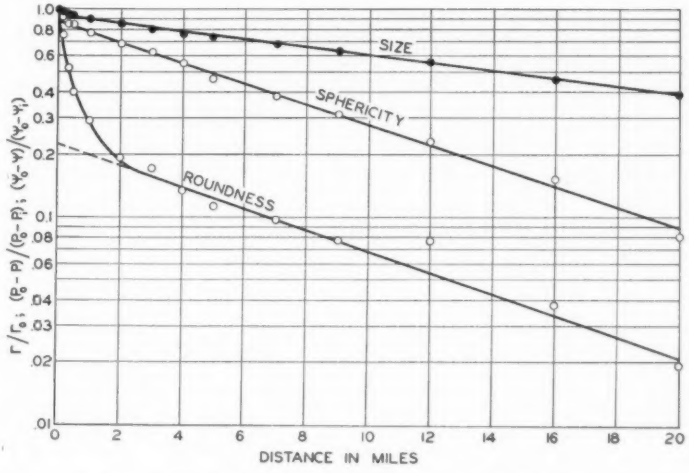

FIG. 5.—Graphic test of abrasion theory (data from Table 2)

metic scale, the graphs will be straight lines descending to the right. Hence, if the theory holds, the experimental data plotted in a similar manner should yield straight lines, with slopes proportionate to the respective values of their coefficients. Figure 5 shows the result, based on Table 2. In compiling the table, values of r_0, P_i, P_0, Ψ_i, and Ψ_0 were chosen from the experimental data. The initial weight, 155 gm., was taken as r_0, and the values r/r_0 in Table 2 represent the ratios of successive weights to this intial weight. The initial roundness, P_i, was taken as 0.13 from Table 1, and the limiting roundness, P_0, was chosen as 0.65, which is 0.01 larger than the final observed value of roundness. Similarly, Ψ_i was chosen as 0.65, to correspond

with the initial sphericity, and Ψ_i was taken as 0.78, which is 0.01 larger than the final sphericity value observed.

It is obvious that the points shown on Figure 5 do not fall on straight lines throughout their entire extent. Both the size and shape curves show an initial departure from a straight line, but after the first half-mile the agreement with theory is good. Roundness, on the other hand, shows a marked departure from a straight line for

TABLE 2

TRANSFORMED SIZE, ROUNDNESS, AND SPHERICITY DATA ON LIMESTONE FRAGMENTS

DISTANCE IN MILES s	SIZE (WEIGHT IN GRAMS)	ROUNDNESS			SPHERICITY		
	r/r_0	P	P_0-P	$\frac{P_0-P}{P_0-P_i}$	Ψ	$\Psi_0-\Psi$	$\frac{\Psi_0-\Psi}{\Psi_0-\Psi_i}$
0.00	1.00	0.13	0.52	1.00	0.65	0.13	1.00
0.17	0.97	.26	.39	0.75	.66	.12	0.92
0.33	0.95	.38	.27	0.52	.67	.11	0.85
0.50	0.94	.44	.21	0.40	.67	.11	0.85
1.00	0.90	.50	.15	0.29	.68	.10	0.77
2.00	0.85	.55	.10	0.19	.69	.09	0.69
3.00	0.80	.56	.09	0.17	.70	.08	0.62
4.00	0.76	.58	.07	0.135	.71	.07	0.54
5.00	0.73	.59	.06	0.115	.72	.06	0.46
7.00	0.69	.60	.05	0.096	.73	.05	0.38
9.00	0.63	.61	.04	0.077	.74	.04	0.31
12.00	0.55	.61	.04	0.077	.75	.03	0.23
16.00	0.47	.63	.02	0.038	.76	.02	0.15
20.00	0.39	0.64	0.01	0.019	0.77	0.01	0.077

the first 2 or 3 miles, after which the straight line is essentially parallel to the sphericity line. These data permit a tentative acceptance of the theory after an initial period of adjustment. It is necessary to examine the initial departures more closely to see whether they are significant.

The size-reduction curve is a direct statement of Sternberg's law applied to a group of particles instead of to a single particle, and the variable r is expressed as the arithmetic mean pebble weight. Schoklitsch has shown that Sternberg's law applies to single particles under abrasion, but that when breakage occurs the weight departs

from the simple theory. In the present experiment breakage was known to occur during the first half-hour of the experiment, and it seems reasonable to attribute the scattering of points during the initial stage of the experiment to this cause. In similar manner the sphericity curve shows a steepening above the theoretical values during the first half-hour. The essential similarity in the behavior of the size and sphericity data suggests that during breakage the sphericity increases more rapidly than during abrasion. Hence it seems reasonable to attribute the initial departure of the sphericity data to breakage and at present to accept equations (4) and (6) as being essentially satisfied by the observed data.

The roundness curve shows too marked a departure from a simple straight line to attribute the effect to simple breakage, although that presumably enters the results. However, the roundness curve does not become a straight line until after the first 2 miles are passed, and the resulting straight line is practically parallel to the sphericity curve. This suggests that two distinct processes are involved in rounding during abrasion. At the start of the process (even without breakage) the increase in rounding is very rapid; but after the most angular corners are worn down, the remainder of the process takes place at about the same rate as shape change. This suggests that at least two exponential functions are operating in conjunction to produce the observed result. The fact that the roundness curve becomes parallel to the sphericity curve (this effect may also be seen in Fig. 3) indicates that sphericity probably controls the rounding process in the long run. The writer had anticiptaed that both rounding and shape change would be affected directly by size, but within the range of the experiment there is no tendency for either curve to become parallel with the size line in Figure 5. It may be that during prolonged abrasion the particles finally become small enough so that the effect of size enters, but this does not appear to be the case in this experiment.

In order to include the effect of shape on roundness it is necessary to re-express equation (2) in such manner that sphericity enters the results. It is convenient to express roundness as $(P - P_o)$ and sphericity as $(\Psi - \Psi_o)$ in this development, so that the rate of change of rounding with respect to distance becomes $d(P - P_o)/ds$. This rate

of change may be considered proportionate to the sphericity diminished by an initial rapid rounding effect. This yields the differential equation

$$\frac{d(P - P_o)}{ds} = b(\Psi - \Psi_o) - a_2(P - P_o) . \qquad (10)$$

Here b is a constant of proportionality affecting the degree to which sphericity enters the results, and a_2 is a constant of proportionality as in the original expression for rounding, equation (2).

Equation (10) may be solved by first substituting the equivalent to $(\Psi - \Psi_o)$ from equation (6) and then rearranging the terms into the standard form

$$\frac{d(P - P_o)}{ds} + a_2(P - P_o) = b(\Psi_i - \Psi_o)e^{-a_3 s} . \qquad (11)$$

Equation (11) is of the type $dy/dx + Py = Q$, which is described in all textbooks of differential equations.[22] The solution is obtained according to standard methods and is found to be

$$P - P_o = \frac{b(\Psi_i - \Psi_o)}{a_2 - a_3}[e^{-a_3 s} - e^{-a_2 s}] + (P_i - P_o)e^{-a_2 s} ,$$

when the following boundary conditions are included: $P = P_i$ when $s = 0$. This solution can be changed to a more convenient form by changing signs and rearranging:

$$P_o - P = (P_o - P_i)e^{-a_2 s} + \frac{b(\Psi_o - \Psi_i)}{a_2 - a_3}[e^{-a_3 s} - e^{-a_2 s}] . \qquad (12)$$

Although equation (12) appears at first glance to be considerably more complicated than equation (5), inspection shows that it resolves to the former when certain terms are omitted. For example, if the sphericity effect is considered negligible, the second term of equation (12) may be ignored, and the remainder reduces to $(P_o - P)/(P_o - P_i) = e^{-a_2 s}$, which is the form of equation (5).

To test equation (12) it is necessary to evaluate certain constants. The coefficients a_2 and a_3 may be found from the graphs of Figure 5. The initial value of the straight line representing sphericity is 0.86.

[22] D. A. Murray, *Introductory Course in Differential Equations* (New York: Longmans, Green & Co., 1932), pp. 26–27.

Half this value, 0.43, is found at $s = 6$ miles, which accordingly represents the half-distance along the theoretical curve. By dividing the number 0.693 by the half-distance, the coefficient a_3 can be found:[23] $a_3 = 0.693/6 = 0.11$.

To find a_2 it is necessary to remove the effect of the sphericity curve from the roundness curve over the steep initial part of the

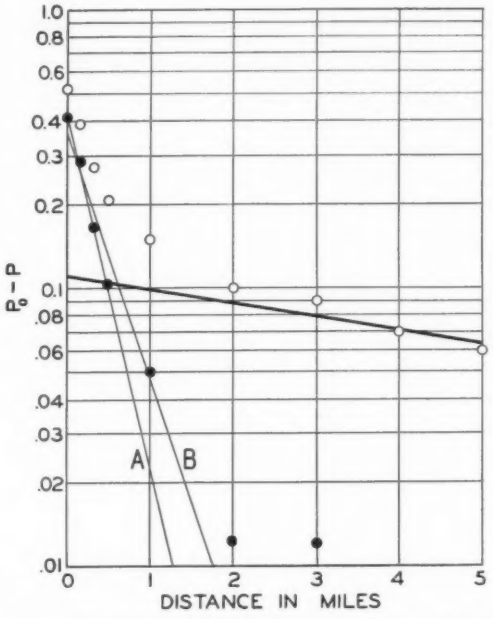

FIG. 6.—Graphic method for segregating a_2 and a_3. The average of the steep lines A and B represents the "difference curve" of rounding and sphericity.

latter. This process is shown in detail in Figure 6, which represents the first 5 miles of the experiment.[24] The circles show the observed values of $(P_0 - P)$ taken from Table 2. The heavy solid line represents the sphericity graph of Figure 5, translated parallel to itself

[23] Krumbein and Pettijohn, *op. cit.*, pp. 208 ff.

[24] In this figure the ordinates are expressed as $(P_0 - P)$ in accordance with eq. (12). In Fig. 5 the roundness data are shown as $(P_0 - P) / (P_0 - P_i)$. The change in ordinates merely shifts the position of the curve without changing its slope.

(with allowances for change of scale) so that it lies approximately along the observed $(P_o - P)$ values in the range beyond 2 miles. At each observed value of $(P_o - P)$ the vertical scale distance between the roundness value and the sphericity line is read. Thus at $s = 0$, the observed $(P_o - P)$ value is 0.52. The translated sphericity line cuts the vertical axis at 0.11. The difference, $0.52 - 0.11 = 0.41$, is plotted as a black dot on the vertical scale. In similar fashion the observed differences of all observations in the steeper parts of the curve are determined. The black dots lie along an approximately straight line; but as the observed $(P_o - P)$ values approach the sphericity line, their values tend to scatter, reflecting the experimental deviations as the effects of initial rounding decrease. Two straight lines have been drawn through the black dots, representing (A) a weighting of the more steeply inclined points at the start of the experiment and (B) a heavier weighting of the values at 1 and 2 miles. It is likely that initial breakage affected the first two or three readings, whereas this effect was diminished after the first half-mile. The half-distances of curves A and B are 0.3 mile and 0.35 mile, respectively. This yields the coefficients 2.31 and 1.98, respectively. The average value is 2.15, which will be accepted as an approximation of a_2, the coefficient of rounding.

The graphic method yields experimental values of a_2 and a_3 but affords no direct method of evaluating the b in equation (12). However, the entire term $b(\Psi_o - \Psi_i)/(a_2 - a_3)$ can be found graphically by extending the straight line which represents roundness in Figure 6 to the point where it intersects the vertical y-axis. This is found to be 0.12. The basis of the graphic method is that, after the effects of a_2 (the rounding coefficient) have become negligible, continued abrasion follows according to the value of a_3, the sphericity coefficient. Thus in equation (12), when the $e^{-a_2 s}$ terms approach zero $(P_o - P)$ approaches $[b(\Psi_o - \Psi_i)/(a_2 - a_3)][e^{-a_3 s}]$. Hence, by extrapolating the straight line to $s = 0$, the value of $b(\Psi_o - \Psi_i)/(a_2 - a_3)$ can be found directly.

From the various values found experimentally (Table 2) and graphically, equation (12) may now be written

$$0.65 - P = 0.52e^{-2.15s} + 0.12(e^{-0.11s} - e^{-2.15s}) . \qquad (13)$$

Equation (13) has all items evaluated except P and s. Hence by choosing values of s, corresponding theoretical values of P may be found. This has been done in Table 3, which shows the calculated and observed values of P together with the relative error between the two. Figure 7 is a graph which indicates the extent of the agreement between the two sets of values. An analysis of the error column of Table 3 shows that the largest errors occur over the first half-mile, which corresponds to that portion of the size and sphe-

TABLE 3

COMPARISON OF OBSERVED AND CALCULATED ROUNDNESS OF LIMESTONE FRAGMENTS

Distance in Miles s	Observed P	Calculated P	Relative Error (Per Cent)
0.00	0.13	0.13	0.0
0.17	.26	.25	3.8
0.33	.38	.34	10.5
0.50	.44	.45	2.3
1.00	.50	.49	2.0
2.00	.55	.55	0.0
3.00	.56	.56	0.0
4.00	.58	.57	1.7
5.00	.59	.58	1.7
7.00	.60	.59	1.7
9.00	.61	.61	0.0
12.00	.61	.62	1.6
16.00	.63	.63	0.0
20.00	0.64	0.64	0.0

ricity data which were affected by breakage. These departures are accordingly assigned to the same cause.

The agreement between theory and observation in the experimental data suggests that rounding is a process which depends to a large extent on the shape of the particles, as equation (10) assumed. Thus the resulting equation (12) should afford some insight into the dynamical process of rounding. In this connection the $b(\Psi_0 - \Psi_s)/(a_2 - a_3)$ coefficient is instructive. Analysis of that coefficient shows that it is the difference between initial and final sphericity which largely controls the process; for given values of b, a_2, and a_3, the larger the sphericity change, the larger the total coefficient, and hence the higher the roundness at any point along the graph.

Similarly, the greater the difference between a_2 and a_3 (the rounding and sphericity coefficients), the more rapidly will roundness increase. Finally, the larger the value of b, the greater the rounding. The constant b has no immediately identifiable physical meaning, although its dimensions are 1/distance. In the present experiment its value is 1.73, but in the absence of additional experimental data little can be predicted about its behavior.

As a summary of the abrasion theory, it may be stated that size reduction appears to be a simple negative exponential function of

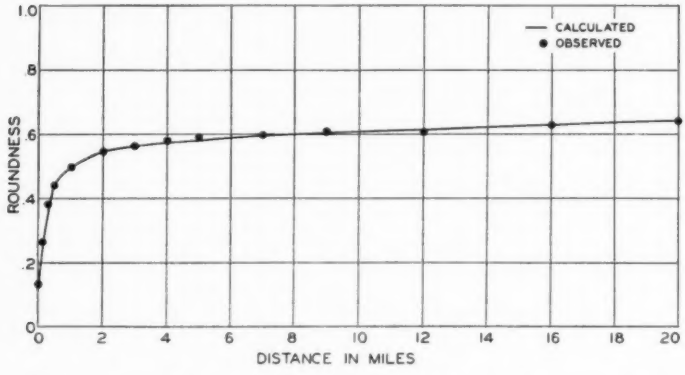

FIG. 7.—Theoretical curve based on equation (13), showing agreement between observed and calculated values of P.

the type shown in equations (4) and (7); sphericity-change is a negative exponential when expressed in the form shown in equation (6); in the form given in equation (9) it is expressed in such a manner that the curve rises.[25] Both equations (6) and (9) are identical, of course; the form of the curve merely depends upon whether sphericity is expressed directly as Ψ along the ordinate or whether the

[25] Since this paper was written, Thiel (*Jour. Sed. Pet.*, Vol. X [1940], pp. 103–24) published the results of abrasion studies on mineral grains and included some data on sphericity. Graphs of the data show that some minerals apparently behave as in the writer's experiment, but others yield S-shaped curves. For the latter it may be necessary to use an equation of the type $d\Psi / ds = c_1\Psi + c_2\Psi^2$ instead of eq. (3). This complicates the function by introducing the second power of the sphericity, which is equivalent to using an additional term in the Taylor expansion (see appendix).

term $(\Psi_0 - \Psi)/(\Psi_0 - \Psi_i)$ is plotted against distance. The expression for rounding given in equations (5) and (8) is a rough approximation of the function if either the initial departure is ignored or if the long-time trend is ignored. Depending upon which part of the graph is used, different values of the rounding coefficient will be found. A closer approximation to rounding is given by equation

TABLE 4

DIMENSIONS OF QUANTITIES IN ELEMENTARY THEORY OF ABRASION

Quantity	Definition	Dimension with Distance as Independent Variable
r	Size expressed as millimeters, mass, etc.	Length (mass, etc.)
r_0	Initial size	Length (mass, etc.)
r/r_0	Ratio of size to initial size	Nondimensional
P	Roundness (Wadell)	Nondimensional
P_i	Initial roundness	Nondimensional
P_0	Limiting roundness	Nondimensional
Ψ	Sphericity (Wadell)	Nondimensional
Ψ_i	Initial sphericity	Nondimensional
Ψ_0	Limiting sphericity	Nondimensional
a_1	Coefficient of size reduction	1/distance
a_2	Coefficient of rounding	1/distance
a_3	Coefficient of shape change	1/distance
b	Shape coefficient in rounding	1/distance
a_2/a_1	Ratio of rounding to size reduction	Nondimensional
a_3/a_1	Ratio of shape change to size reduction	Nondimensional
a_2/a_3	Ratio of rounding to shape change	Nondimensional

(12), which is an exponential function involving the simultaneous operation of two negative exponentials, one of which dominates at the start and the other of which controls the long-time aspects of rounding. To round out the analysis Table 4 is included to show the dimensions of each of the variables and parameters in the analysis. It will be seen that the a's and the b all have the same dimensions and that the ratios a_2/a_1 and the like are nondimensional. Thus the several parameters may be compared directly, and in terms of ratios they afford nondimensional concepts which may be used for further investigations.

The coefficients found in the present experiment are $a_1 = 0.043$, $a_2 = 2.15$, and $a_3 = 0.11$. Hence, the following ratios may be set up to describe the relative rates of size reduction, shape change, and rounding:

$$\text{Rounding : Size} = a_2/a_1 = 2.15/0.043 = 50$$

$$\text{Shape change : Size} = a_3/a_1 = 0.11/0.043 = 2.5.$$

$$\text{Rounding : Shape change} = a_2/a_3 = 2.15/0.11 = 19.5\,.$$

These several ratios correspond to the empirical ratios k_2/k_1, etc., discussed earlier. The precise values differ because the effect of breakage is partially corrected in the later coefficients, and the theoretical development permitted the separation of the rounding coefficient into two values. The earlier reasoning thus leads to essentially the same conclusions, except that the theoretical analysis sheds additional light on the physical processes which occur during abrasion.

GEOLOGICAL APPLICATIONS OF THE ABRASION THEORY

The analytical theory of abrasion appears to be valid for tumbling-barrel data within the range of present experimental data, but much more important geologically is the question whether the findings may be applied to actual field situations. It is doubtful whether the numerical values of the coefficients obtained in tumbling barrels can be applied directly to any field problem, because the numbers obtained depend upon the particular equipment used as well as upon the special conditions of the experiment. Despite this limitation, it seems likely that the general dynamical processes which occur during abrasion in streams and in barrels are similar. That is, the nature of the several functions should be the same, although the exact values of the parameters may change. The theoretical and experimental finding that rounding is a function of itself and of shape change, for example, should apply equally well to streams and to barrels. Moreover, the relative rates of rounding, shape change, and size reduction should hold in the sense that the several ratios discussed earlier will occur in the same order in the field and in the laboratory.

Although the experimental data showed no apparent effect of size on either sphericity change or rounding, this does not mean that size may not be an influence in these processes. Further experimental data are needed in which two or more sets of particles are used, similar in all respects except in initial size and abraded under the same conditions. By studying the variations in the several parameters in terms of the original sizes used, information may be had on the effect of initial size on the rates of change involved.

Field data support the conclusion that initial size is a factor in the rate of rounding. R. D. Russell and R. E. Taylor,[26] for example,

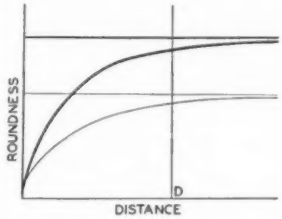

FIG. 8

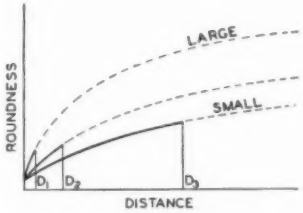

FIG. 9

FIG. 8.—Hypothetical curves of roundness as functions of distance for large and small particles. Selective transport is not a factor.

FIG. 9.—Hypothetical curves of roundness as functions of distance for large, intermediate, and small particles. Selective transport is a factor.

found that samples of Mississippi River sand showed a lower roundness in the smaller size grades than in the larger size grades; they also cite other investigators who reported the same effect. This suggests that either or both of the parameters P_0 and a_2 in equation (12) may be a function of size, such that they increase as initial size increases. On this basis a hypothetical situation may be developed from equation (12) to account for the observations. Figure 8 shows two curves of roundness as functions of distance. The upper heavy curve represents large particles within a given sample, the lower curve represents smaller ones, both starting with low roundness. The large particles become more rounded more rapidly than the small ones. Thus at some point D along the stream a given sample shows

[26] "Roundness and Shape of Mississippi River Sands," *Jour. Geol.*, Vol. XLV (1937), pp. 225–67.

a greater degree of rounding on the larger particles. Hence a graph of roundness against size for all size grades of a sample may well show statistically that the larger particles are the better rounded.

The relative simplicity of the above relation between average roundness and average size may be much modified by selective transportation. If both large and small particles are carried along by the stream, the relation shown in Figure 8 may apply; if, however, the larger particles are left behind during selective transportation so that the smaller ones continually outstrip them, a different situation may arise, which is sketched in Figure 9. Here three curves of roundness as functions of size are shown, associated with large, intermediate, and small particles. The large particles round rapidly but only travel at distance D_1. The intermediate particles round less rapidly but move a distance D_2. The smaller particles round least rapidly but move a distance D_3. As the figure is drawn, it indicates that each of the successively smaller sizes reaches a higher point on its own rounding curve than the preceding ones, thus establishing a net increase in roundness downstream. (The value of the roundness at the several points is indicated by the heights of the ordinates D_1, D_2, and D_3.) If samples are collected at the three points shown and a graph prepared of size against roundness, the graph may show statistically that the smaller particles are the most highly rounded. As far as the writer is aware, Wentworth published the only graph of this type to represent conditions in the field.[27] Wentworth's original roundness data were "corrected" to a standard distance before the graph was prepared, and he does not furnish any independent size data. Hence it is not possible to analyze the situation in detail, but it seems not unlikely that under conditions of selective transport such relations may arise.

In his study of Russell Fork, Wentworth also published a graph showing his final corrected data on quartzite roundness as a function of distance along the stream. It may be recalled that equation (12) is based on the assumption that the particles are of one rock type and that all start with uniform size, or at least that the mean size may be used as an expression for the distribution. Wentworth corrected his data for the size factor, so that presumably the field data were transformed into a set of data to which the abrasion theory may be ap-

[27] "A Field Study . . . . ," *op. cit.*

plied. In order to see whether equation (12) yields a satisfactory smooth curve for the corrected data, Wentworth's values were read from his graph and replotted in terms of an assumed asymptote of 0.150. This chosen asymptote is

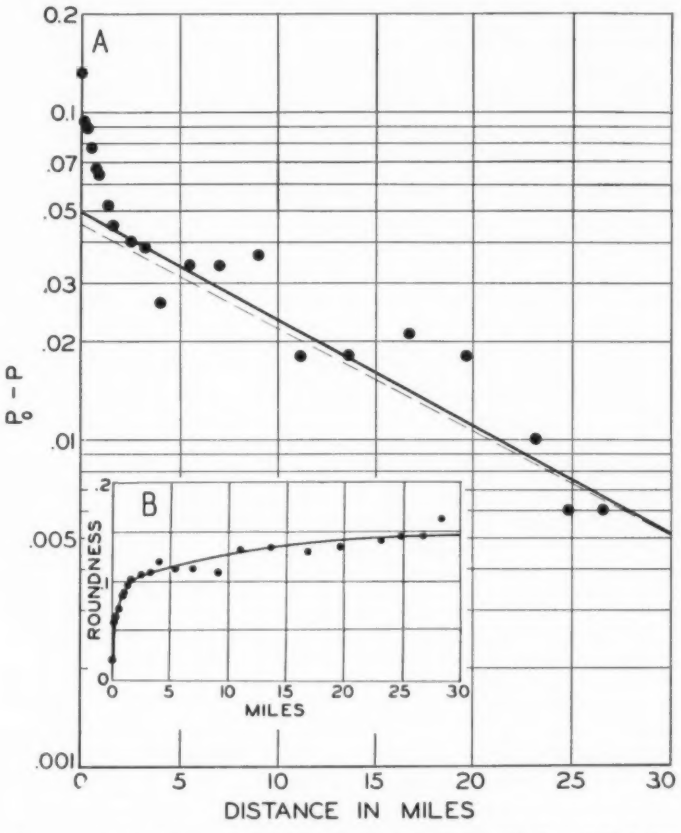

FIG. 10.—*A*, graphic test of abrasion theory with Wentworth's Russell Fork data; *B*, relation between theoretical curve of equation (14) and the observed values.

lower than the last point on Wentworth's graph, but it agrees with the general trend. Figure 10, *A*, shows the replotted data with $(P_o - P)$ along the logarithmic *y*-axis and distance along the arithmetic *x*-axis. At $s = 0$, for example, $P = 0.018$, so that $P_o - P = 0.132$. The points scatter rather widely, but a general

resemblance to Figure 5 may be seen in that the trend decreases in slope with distance. A heavy straight line through the points indicates the approximate average trend.

Wentworth gives no shape data which may be used to obtain the coefficient a_3; but by reasoning in a manner similar to that used in connection with Figure 6, a first approximation to the necessary values may be had. One may assume that the change in shape along Russell Fork was less marked than the change in roundness which would presumably yield a straight line of slope less than the line representing roundness. Following the procedure of Figure 6, such a sphericity line was drawn in Figure 10, *A*, as a dashed line. The scale difference between each value of $(P_o - P)$ and this dashed line was used to draw a "difference curve" as in Figure 6. Half-values based on the sphericity line and on the difference curve yielded the values $a_3 = 0.08$ and $a_2 = 1.73$. The general intercept $b(\Psi_o - \Psi_i)/(a_2 - a_3)$ was found to be 0.05 when the average roundness line was extrapolated to the *y*-axis. These numerical data were used in equation (12) to calculate the theoretical roundness values. The expression assumed the form:

$$P_o - P = 0.15e^{-1.73s} + 0.05(e^{-0.08s} - e^{-1.73s}) . \qquad (14)$$

Table 5 shows the values of *P* calculated from this equation at various distances. In the second column the observed values are given as they were read from Wentworth's graph. The general agreement is quite satisfactory; the mean relative error is only 5.5 per cent, and the largest errors occur in the early stages of rounding. Figure 10, *B*, shows the theoretical curve calculated from equation (14) and the observed points. The general agreement of the two suggests that the nature of the rounding function is approximately the same in the laboratory and in the field. In this connection Wentworth drew a straight line through the latter part of his empirical curve; the theoretical curve of Figure 10, *B*, suggests the asymptotic nature of rounding.

Both Russell and Taylor's and Wentworth's data emphasize the fact that a study of abrasion phenomena alone may not be sufficient to find unique answers to problems encountered in the field. A fuller understanding of the laws of selective transport as they are related to specific properties of particles is required. At present, for example, it would appear from Wentworth's size-roundness data that selective transport may have been present along Russell Fork; the agreement of his final roundness data with the abrasion theory suggests that his size corrections eliminated any such effects. It is clear, however, that these inferences are gratuitous, because the original size data are not given and because the laws of selective transport are not fully known.

Another difficulty involved in any application of the abrasion theory to natural conditions arises from the fact that impact breakage and crushing may yield results markedly different from simple abrasion. For example, during breakage the rate of size decrease may be more rapid than shape change or rounding, thus changing

TABLE 5

COMPARISON OF OBSERVED AND CALCULATED ROUNDNESS OF QUARTZITE PEBBLES*

Distance in Miles s	Observed r_1/R	Calculated r_1/R	Relative Error (Per Cent)
0.0	0.018	0.018	0.0
0.1	.057	.032	43.7
0.3	.061	.052	14.8
0.6	.073	.074	1.4
0.8	.083	.082	1.2
0.9	.085	.086	1.2
1.3	.098	.096	2.0
1.6	.105	.101	3.8
2.5	.110	.108	1.8
3.2	.112	.112	0.0
4.0	.124	.114	8.0
5.5	.116	.118	1.7
7.0	.116	.121	4.3
9.0	.112	.125	11.6
11.2	.132	.129	2.3
13.6	.132	.133	0.8
16.8	.128	.137	7.0
19.7	.132	.139	5.3
23.1	.140	.142	1.4
24.8	.144	.143	0.7
26.6	.144	.144	0.0
28.4	0.163	0.145	11.0

* Data in first two columns read from Fig. 41 of Wentworth, "A Field Study . . . ," *op. cit.*

the relative orders of the several coefficients. The specific effects must probably be determined by experiment, but in special cases the results may be predicted. The crushing of small grains by pebbles may be expected to accelerate the rounding of the pebbles and to decrease the roundness of the crushed particles. Thus, if several processes go on simultaneously or even successively in nature, the final curves of size, shape, and roundness as functions of distance may be quite complex. As with abrasion, however, these several effects may be isolated and studied experimentally.

SUMMARY

The foregoing discussion demonstrates, among other things, that the process of abrasion, even when reduced to its simplest terms, requires rather advanced methods of analysis. Moreover, the elementary theory itself indicates the presence of large gaps in current understanding of abrasion phenomena. For example, the exact effects of initial size on rounding and the factors which control the limiting values which roundness and sphericity approach under given conditions are virtually unknown. The theory does indicate some of the directions in which experimentation may profitably proceed.

The restrictions imposed on the analytical treatment of abrasion also limit the application of the theory to natural conditions. Geologists are interested in such questions as the source of a sediment, the distance of transport, the agent of transport, the conditions at the site of deposition, and the general environment of deposition. Abrasion studies are related mainly to questions of distance of transport. Lithologic composition is important in determining the source of sediments. Size and shape are related to the agent and conditions of transportation, because these factors in turn largely control settling velocity. This suggests that a critical study of particle characteristics which are affected more strongly by one process or another may yield criteria for field interpretation.

APPENDIX

The theory of abrasion as developed in the text is a restricted form of a more general approach to the problem. In order to round out the discussion, some of the factors involved in the more general treatment will be sketched here.

In terms of earlier work, and particularly of research during the last decade, it is possible to re-express Wentworth's earlier thesis on abrasion as follows:

The fundamental characteristics of particles which may change during abrasion are size, sphericity, roundness, and surface texture. The rate of change of any one of these four variables with respect to distance may be a function of itself and of the other three variables, as well as of lithologic composition and of the nature and kind of motion to which the particles are subjected. In an experimental sense the physical constants of the tumbling barrel and the absolute size of the load may also influence the results.

In the present approach to abrasion theory the surface texture was ignored, largely because there are no satisfactory methods available for expressing it

quantitatively. Further, by using limestone fragments of restricted size, shape, and roundness, subjected to constant rigor during abrasion, a number of variables were held constant, and the resulting equations were relatively straightforward. Thus such quantities as a_1, a_2, and a_3 proved to be constants, as were P_0 and Ψ_0. In the more general approach these quantities would be parameters; that is, the value assumed by them depends upon the nature of particular experiments. Just how many factors, such as initial size, initial shape, rock type, rigor, and others, may affect the shape coefficient a_3, for example, is not known. These factors may be evaluated by experiments in which one after another of the variables is changed, so that the nature of each function may be discerned. Until more data accumulate, it seems premature to develop analytical expressions for these functions. Without some knowledge of the behavior of each variable the resulting expressions are too general for direct application. Moreover, if all possible variables are included in the most general approach, the number of arbitrary constants and coefficients increases rapidly. This may be illustrated by the following brief outline of such a general approach, based on the use of Taylor's theorem.

The variables involved in a general approach to abrasion theory are:

Size, r	Rock type, R
Roundness, P	Rigor of the process, H
Sphericity, Ψ	Distance, s
Surface texture, S	

What is desired is an analytical expression relating each of the first four variables to the last variable, in terms of itself and the others, as modified by rock type and rigor. With no experimental data whatever, one would assume that the rate of change of size with respect to distance is some function of itself and the other variables:

$$\frac{dr}{ds} = f_1(r, P, \Psi, S, R, H) .$$

In similar manner roundness, sphericity, and surface texture could be expressed by such an equation. The resulting functions are continuous and analytic, so that each one may be expanded by Taylor's theorem as follows:

$$\frac{dr}{ds} = m_0 + m_1 r + m_2 P + m_3 \Psi + m_4 S + m_5 R + m_6 H .$$

Corresponding expressions for roundness, sphericity, and surface texture would yield three more equations, with a total of twenty-eight constants of the type m_i. The resulting equations could be handled in terms of their characteristic equation, which would yield a twenty-eighth-order determinant. Clearly, the analysis would soon extend beyond the bounds of the immediately practical in

terms of field data on sediments. This is not to belittle such an approach;[28] on the contrary, if sufficient experimental data are available to eliminate some of the m's, the method furnishes a powerful tool for analysis. For example, in the present experiment, rock type and rigor were held constant, and the effect of surface texture was ignored. Thus the restricted theory permitted the evaluation of some of the constants as follows: $m_0 = 0$; $m_1 = -a_1$; $m_2 = 0$; $m_3 = 0$. Thus, by a single experiment the complexity of the general expression is much reduced. In similar fashion other experiments would permit the evaluation of the other constants.

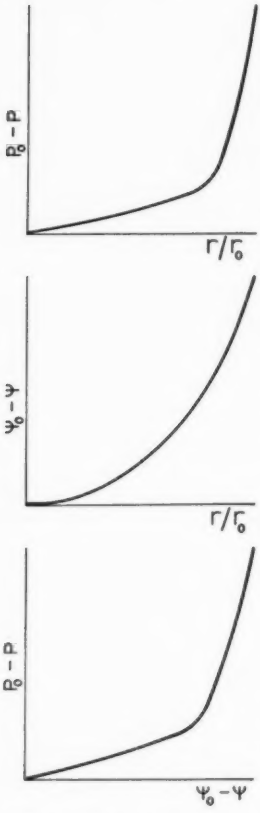

FIG. 11.—Graphs of equations (15)–(17), showing interrelations among size, sphericity, and roundness during abrasion.

Certain complexities in the rounding process, such as Wentworth's experimental finding that the roundness of marble cubes may decrease after reaching a maximum, may be derived from the general theory of abrasion. Such effects may depend upon certain combinations of exponents in functions which involve the simultaneous operation of two or more exponentials. Physically it is possible that such effects may be due simply to initial differences in the edges of the cubes. Such initial differences in length are negligible until the particle is close to its asymptotic shape and roundness values, whereupon slight differences in axial lengths may manifest themselves by a statistically preferred orientation during rolling. That is, if there is a tendency for the center of gravity of the particle to remain at its lowest point during movement of the particle, this itself would ultimately result in an increasing departure from true sphericity. A slightly prolate spheroid would tend to roll with its longer axis essentially horizontal, so that during further abrasion the ratio of the long axis to the shorter axes would increase.

In the present paper only four variables were used: size, sphericity, roundness, and distance. Three functional relations were developed in the text, but by the theory of equa-

[28] An excellent discussion of this method, applied to biological problems, is given in A. J. Lotka, *Elements of Physical Biology* (Baltimore: Williams & Wilkins, 1925), chaps. vi–viii.

tions it should be possible to set up six analytical expressions among these four variables. These are:

Size as a function of distance
Roundness as a function of distance
Sphericity as a function of distance
Roundness as a function of size at any distance s
Sphericity as a function of size at any distance s
Roundness as a function of sphericity at any distance s

The first three of these relations have been given in the text; the last three may be had by eliminating ds from successive pairs of the three differential equations (1), (3), and (10) of the text. This process yields three additional differential equations. Their solution is somewhat tedious, although it presents no difficulties, inasmuch as all of the equations are linear and of first order and first degree. The final solutions are given here for completeness:

$$P_o - P = [(P_o - P_i) - K]\left[\frac{r}{r_o}\right]^{a_2/a_1} + K\left[\frac{r}{r_o}\right]^{a_3/a_1}, \tag{15}$$

$$\Psi_o - \Psi = [\Psi_o - \Psi_i]\left[\frac{r}{r_o}\right]^{a_3/a_1}, \tag{16}$$

$$P_o - P = [(P_o - P_i) - K]\left[\frac{\Psi_o - \Psi}{\Psi_o - \Psi_i}\right]^{a_2/a_3} + K\left[\frac{\Psi_o - \Psi}{\Psi_o - \Psi_i}\right]. \tag{17}$$

Here $K = b(\Psi_o - \Psi_i)/(a_2 - a_3)$. It will be noted that K is the intercept mentioned in connection with equation (12). Equation (15) covers the relation between roundness and size at any point s; equation (16) shows the relation between sphericity and size at any point s; and equation (17) covers the relation between roundness and sphericity at any point s.

Detailed discussion of the implications of these derived relationships is deferred until more experimental data are available. It may be mentioned in passing, however, that inasmuch as the present experimental data showed $a_2 > a_3 > a_1$ the several ratios of the coefficients in equations (15)–(17) are all larger than 1. Under these conditions the equations have the general forms shown in Figure 11. It is interesting to note that all three relations are power functions rather than exponential functions of the type already encountered. Among these three functions equation (16) plots as a simple straight line of slope a_3/a_1 on double log paper. The other two functions plot as double line segments on double log paper, because they both consist of the sums of two power functions. Hence their graphs (Fig. 11) show a more marked change in trend than the simple power function of equation (16).

ACKNOWLEDGMENTS.—The writer is indebted to several men who have shown interest in the problems raised by this study. Dr. Carl Eckart, of the department of physics of the University of Chicago, gave many helpful suggestions regarding the analytical treatment of the problem. Dr. R. G. Sanger, of the department of mathematics, checked the rigor of the mathematical derivation of equations (15)–(17). Dr. F. J. Pettijohn, of the department of geology, read the manuscript for its geological implications.

TOPOGRAPHIC ANALYSIS OF THE MONTEREY STAUNTON, AND HARRISONBURG QUADRANGLES

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ABSTRACT

This paper includes a discussion of altimetric curves, hypsographic curves, and tabulated data on the relation of topography to bedrock. Interpretation of the curves and other data leads to the following conclusions: (1) Within the limits of weak rocks the Harrisburg peneplain is well developed in the Harrisonburg quadrangle, fairly well developed in the Staunton quadrangle, and poorly developed in the Monterey quadrangle. (2) No peneplain above the Harrisburg is evidenced by the methods here used. Lack of clear evidence of the Schooley and/or other upland peneplains in this region indicates either that (*a*) they were never developed here or that (*b*) they have been destroyed by subsequent erosion. (3) Considering the close relation between topography and bedrock and the inevitable truncation of deformed strata by long-continued erosion without peneplanation, the present topography can be satisfactorily explained without recourse to former planation of weak and resistant rock alike.

INTRODUCTION

The Appalachian Mountains have long been a fruitful field of geomorphic investigation; yet no one can say that the final word in the erosional history of that interesting region has been written. Differences of opinion exist among geologists as to the meaning of many of the topographic forms. Since Davis developed the concept of the cycle, many investigators assign cyclic significance to accordant summits, flat tops, even crest lines, shoulders, benches, and other breaks in slope; others think these features can be explained by noncyclic causes. Differences of interpretation exist regarding the number and degree of perfection of the peneplains represented in the Appalachian Mountains. In the region here discussed G. W. Stose and H. D. Miser[1] see four separate peneplains—valley floor, intermediate, upland, and summit peneplains. F. J. Wright[2] recognizes only two peneplains—the valley floor (Harrisburg) and the upland (Schooley). Wright further states that the Schooley pene-

[1] "Manganese Deposits of Western Virginia," *Va. Geol. Surv. Bull. 23* (1922), pp. 16–24.

[2] "The Newer Appalachians of the South, Part I: Between the Potomac and New Rivers," *Denison Univ. Bull.*, Vol. XXXIV, No. 13 (*Jour Sci. Lab.*, Vol. XXIX [1934], p. 6).

plain has been so largely destroyed by subsequent erosion that the probable error in estimating its present level is several hundred feet.[3] In a recent paper J. L. Rich[4] analyzes the commonly used criteria of peneplanation and concludes that several of them have no cyclic significance. G. H. Ashley[5] in Pennsylvania also has questioned some of the peneplain criteria. He points out the importance of structure, gives evidence of the rapid rate of denudation, and emphasizes the effects of differential erosion on the topography.

This contribution, which includes a somewhat different method of attack, deals with a relatively small region in some detail but by no means exhaustively. Multiple erosion surfaces are not presupposed, and peneplains are recognized only where the topography cannot otherwise be readily interpreted. It is probably too much to expect that criteria will eventually be established whereby peneplains may be identified as objectively and as confidently as are rock formations, but work toward that end should be fruitful.

Except where otherwise stated, the region here discussed is included in the Monterey (Virginia–West Virginia), Staunton (Virginia–West Virginia), and Harrisonburg (Virginia), 30-minute quadrangles (Fig. 1). These quadrangles extend from the Appalachian Plateau in the northwestern part of the Monterey quadrangle across the Ridge and Valley and the Blue Ridge provinces to the Piedmont in the southeastern corner of the Harrisonburg quadrangle. Since this region, as well as adjacent portions of the Appalachian Mountains, has been discussed by a number of writers, no general description is here given. The reader is especially referred to U.S. Geological Survey Folios, Nos. 14 and 61, of the Staunton and Monterey quadrangles, respectively, and to two excellent comprehensive papers by Wright.[6] In the 1925 paper Wright gives a review of previous literature on the region.

[3] *Ibid.*, pp. 14–15.

[4] "Recognition and Significance of Multiple Erosion Surfaces," *Bull. Geol. Soc. Amer.*, Vol. XLIX (1938), pp. 1695–1722.

[5] "Studies in Appalachian Sculpture," *Bull. Geol. Soc. Amer.*, Vol. XLVI (1935), pp. 1395–1436.

[6] "The Physiography of the Upper James River Basin in Virginia," *Va. Geol. Surv. Bull. 11* (1925); and "The Newer Appalachians of the South . . . . ," *op. cit.*

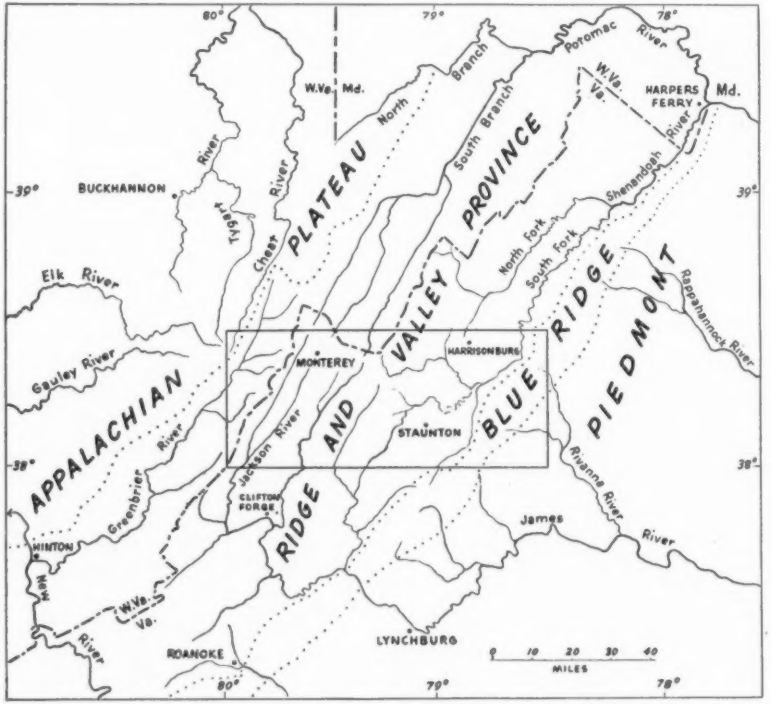

FIG. 1.—Index map

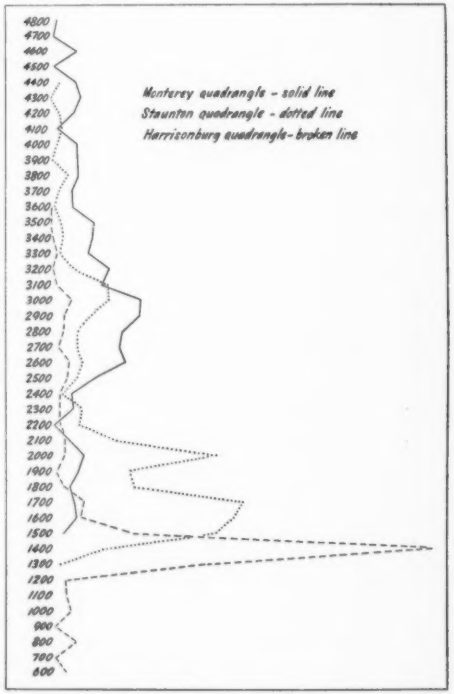

FIG. 2.—Altimetric curves of the Monterey, Staunton, and Harrisonburg quadrangles.

SUMMITS

SIGNIFICANCE OF SUMMITS

In the Appalachian Mountains, as in many other regions, summits have long been considered as having cyclic significance. Accordant summits are used as an important criterion for the location of elevated peneplains. Some writers consider accordant summits as actual remnants of an ancient uplifted peneplain surface; others judge such summits to be portions of a peneplain uniformly reduced from a higher altitude; while some think that accordant summits may have little or no cyclic significance. Discordant summits are also variously interpreted. To some they represent a peneplain that was originally imperfect or has been made so by warping, faulting, or differential lowering; to others they have no cyclic meaning. Flat summits are viewed by some as strong evidence of earlier peneplanation, while others think that such summits reflect the structure in noncyclic erosion.

Imperfect methods of observation and portrayal no doubt play a considerable part in the prevalent disagreement as to the meaning of summits. Views from hilltops, long-range ground photographs, and projected profiles are methods commonly employed. Ground photographs, like eye-views, though often very instructive, are apt to produce optical illusions. Views of the same region from different stations may have markedly different aspects. Aerial photographs usually give better perspective to long-range views than those taken from the ground. Projected profiles, as pointed out by Rich,[7] unduly emphasize the appearance of horizontality; another disadvantage is the large space required to publish them.

Preponderance of summits at certain altitudes does not necessarily indicate peneplanation at those levels; there are other conditions that determine the altimetric distribution of summits. Under certain conditions the summits in a region should be most numerous at an altitude about midway between the altitudes of the highest and lowest summits. Such conditions would consist of (1) rocks of many different degrees of resistance well distributed throughout the region and (2) erosion in one cycle until all parts of the original surface

[7] *Op. cit.*, p. 1699.

have been destroyed and the main streams have reached grade—the mature stage of topography. Under these conditions summits on the weakest rock would range from the lowest altitude near the main streams, up to moderate altitude away from the main streams or where protected by resistant rock; summits on the most resistant rock would range from the highest altitude where distant from the main streams, down to moderate altitude where especially exposed; summits on the rocks of intermediate resistance would range from somewhat above to somewhat below moderate elevation. Thus, the number of summits would be few at both extremes of altitude and most numerous at intermediate altitudes. Any departure from the conditions stated above, either in the rocks or in the stage of erosion, should be reflected in the altimetric distribution of summits. A region that has not yet reached the mature stage should have a preponderance of summits at the upland altitude if the original surface were essentially flat. A region well past early maturity should have broad summits at low altitude. A region with two types of rock differing greatly in resistance and no rocks of intermediate resistance should, throughout much of the cycle, have most of the summits near the two extremes of altitude and few in between. If there are several rock types, each differing sharply in resistance from adjacent ones, summit maxima may occur at several different altitudes. Distribution of the various rock types resulting from structure will have a marked influence on the altimetric distribution of summits. Thus, there are circumstances of rock resistance and distribution and stage of erosion that may combine to produce a preponderance of summits at certain altitudes irrespective of former peneplanation.

ALTIMETRIC CURVES OF SUMMITS

The altimetric curve (Fig. 2), as here constructed, shows the altitudinal distribution and areas of all summits. In 1936 the writer[8] used a similar device to depict summits in the West Point and Schunemunk quadrangles, New York. According to S. W. Wooldridge and R. S. Morgan,[9] a curve of similar nature for spot eleva-

[8] "Hudson Gorge in the Highlands," *Bull. Geol. Soc. Amer.*, Vol. XLVII (1936), No. 12, p. 1842, Fig. 7.

[9] *Outlines of Geomorphology* (*The Physical Basis of Geography*) (New York: Longmans, Green & Co., 1937), pp. 265–66.

tions has been used in France by Baulig. The data for curves, as here used, are obtained by measuring the area enclosed by each summit contour on the topographic maps. The total area of summits at each elevation is then plotted to scale. The degree of accuracy of the curve is dependent on the accuracy of the map and the care with which the summit areas are measured. Large summits can be satisfactorily measured with a planimeter; the smaller ones are more conveniently measured by superposing them onto a grid of closely spaced lines. Carefully made from good maps, the altimetric curve is an objective method of showing the altitudes and areas of summits; of course, it does not explain them. Maxima on the curve show altitudes at which summits are numerous or large, while minima show the altitudes at which summits are few or small. Pronounced maxima, unless otherwise explained, presumably represent planation. Probably the most important defect of this kind of curve is the fact that it does not show the areal distribution of summits. This information must be otherwise supplied.

SUMMITS OF THE MONTEREY QUADRANGLE

Altimetric distribution and area.—The area enclosed by the 658 summit contours is 16.82 square miles, which is about 1.79 per cent of the total area of the quadrangle. As a rule, the higher summits are sharper than those of low altitude.

The curve (Fig. 2) is saw-toothed. To some, this shape may suggest planation at the points of maxima. Thus, planation of greater or lesser amount at ten different levels would be indicated. To me the curve seems to reflect the structure and the different degrees of resistance of the various rock formations. Irregular succession of weak and resistant rocks in this folded region, causing local summit maxima and minima, gives the notched aspect to the curve.

The most pronounced maximum on the altimetric curve of the Monterey quadrangle is the generalized bulge from 2,500 to 3,500 feet. This maximum is nearly midway between the highest and lowest summits, as it should be after long-continued erosion in a region of several different degrees of rock resistance. It consists of summits on most of the formations in the quadrangle, being composed mostly of the rocks of intermediate resistance but also including the higher

summits on the weak rocks and the lower summits on the resistant rocks. As this maximum is considerably below the tops of the high ridges, it is not likely that it represents the Schooley peneplain. Possibly it represents an intermediate peneplain. Most probably it is a normal maximum midway between the highest and lowest summits which has no cyclic significance.

Formations on which summits occur.—Table 1 contains the formations of the Monterey quadrangle with the area (square miles) of outcrop of each and the number of summits occurring at the various altitudes on each. The table does not include the areas of the summits and, therefore, does not correspond exactly with the altimetric curve. Summits at high altitudes are limited to a few formations, those at low altitudes are also limited to a few formations, and those at intermediate altitudes are found on most of the formations, a condition which corresponds with the idea that there should be a normal intermediate maximum of summits in a maturely dissected region of many rock types.

The highest summits, on the Blackwater (Pottsville) formation, are in the flat Cheat Mountain–Allegheny Front syncline. Slightly lower are the summits on the Pocono and Hampshire (Catskill) formations in the synclinal Allegheny Mountain.[10] High summits on the Tuscarora formation are in the large Jack Mountain and Back Creek Mountain anticlines; the basalt intrusion in Sounding Knob is 4,400 feet. Tuscarora, as here used, includes the Juniata, Tuscarora, and Cacapon formations as mapped in the Monterey folio. Lower summits on the Tuscarora are found where the large anticlines plunge and where small anticlines do not raise this formation to high altitude. Some of the formations, especially the Hampshire and the Jennings, which are variable in composition, have summits through a wide range of elevations. The Shenandoah limestone, stratigraphically low and also easily eroded, occurs only in the anticlinal valleys. No summits occur on the Martinsburg and the Canaan (Mauch Chunk) shales and the Greenbrier limestone. These

[10] It should be noted that, in the Monterey folio, Allegheny Mountain is designated the "Allegheny Front," whereas by common agreement the Allegheny Front in this region is now located on Back Allegheny Mountain.

weak formations crop out adjacent to markedly resistant beds—a position not favorable to the development of summits.

TABLE 1

TABULATION OF SUMMITS ON THE FORMATIONS IN THE MONTEREY QUADRANGLE

Altitude of Summits (Feet)	Shenandoah Limestone (45.08 Sq. Mi.)	Martinsburg Shale (21.06 Sq. Mi.)	Tuscarora Sandstone (75.62 Sq. Mi.)	Rockwood Formation (86.39 Sq. Mi.)	Lewistown Limestone (134.43 Sq. Mi.)	Monterey Sandstone (59.94 Sq. Mi.)	Romney Shale (113.06 Sq. Mi.)	Jennings Formation (288.91 Sq. Mi.)	Hampshire Formation (66.65 Sq. Mi.)	Pocono Sandstone (2.38 Sq. Mi.)	Greenbrier Limestone (7.14 Sq. Mi.)	Canaan Formation (15.95 Sq. Mi.)	Blackwater Formation (20.16 Sq. Mi.)	Basalt Intrusion	Totals
4,800	....	....	....	....	....	....	....	....	....	....	....	....	3	....	3
4,700	....	....	....	....	....	....	....	....	....	....	....	....	4	....	4
4,600	....	....	....	....	....	....	....	....	2	4	....	....	3	....	9
4,500	....	....	....	....	....	....	....	....	4	....	....	....	....	....	4
4,400	....	....	3	....	....	....	....	....	6	2	....	....	....	1	12
4,300	....	....	1	....	....	....	....	....	9	....	....	....	1	....	11
4,200	....	....	....	....	....	....	....	2	9	....	....	....	2	....	13
4,100	....	....	3	....	....	....	....	3	2	....	....	....	....	....	8
4,000	....	....	7	....	....	....	....	3	3	....	....	....	....	....	13
3,900	....	....	13	....	....	....	....	5	....	....	....	....	....	....	18
3,800	....	....	17	....	....	....	....	5	....	....	....	....	....	....	22
3,700	....	....	9	....	....	....	....	2	1	....	....	....	....	....	12
3,600	....	....	10	....	....	....	....	2	....	....	....	....	....	....	12
3,500	....	....	3	....	1	....	....	16	....	....	....	....	....	....	20
3,400	....	....	2	1	1	1	....	21	....	....	....	....	....	....	26
3,300	....	....	12	4	3	1	....	12	....	....	....	....	....	....	32
3,200	1	....	5	6	7	8	....	18	2	....	....	....	....	....	47
3,100	1	....	10	5	9	2	....	13	....	1	....	....	....	1	42
3,000	1	....	5	10	8	13	....	6	6	....	....	....	....	....	49
2,900	3	....	5	3	9	6	....	11	9	....	....	....	....	....	46
2,800	1	....	3	7	7	6	2	9	5	....	....	....	....	....	40
2,700	2	....	2	4	17	9	3	10	....	....	....	....	....	....	47
2,600	....	....	1	3	12	8	3	7	....	....	....	....	....	....	34
2,500	....	....	....	2	9	8	2	1	....	....	....	....	....	....	22
2,400	....	....	....	1	2	10	4	....	....	....	....	....	....	....	17
2,300	....	....	....	3	2	9	....	3	....	....	....	....	....	....	17
2,200	....	....	....	....	....	1	....	1	....	....	....	....	....	....	2
2,100	....	....	....	1	....	4	1	3	....	....	....	....	....	....	9
2,000	....	....	....	....	....	1	2	8	....	....	....	....	....	....	11
1,900	....	....	....	1	1	4	1	7	....	....	....	....	....	....	14
1,800	....	....	....	....	....	1	1	12	....	....	....	....	....	....	14
1,700	....	....	....	....	....	1	4	4	....	....	....	....	....	....	9
1,600	....	....	....	....	....	1	7	6	....	....	....	....	....	....	14
1,500	....	....	....	....	....	1	3	1	....	....	....	....	....	....	5
Total	9	....	111	51	88	95	33	191	58	7	....	....	13	2	....
Grand total	....	....	....	....	....	....	....	....	....	....	....	....	....	....	658

In this region, composed mostly of folded rocks, the relative mean elevations of the summits are probably proportionate to the rela-

tive resistance of the formations. The number of summits per unit area of outcrop is another indicator of relative resistance of the formations. By noting the average altitude of the summits and the number of summits per unit area of outcrop on each formation, one might arrive at a fairly accurate scale of relative resistance of the various rocks. On this basis the Tuscarora, Hampshire, Pocono, and Blackwater formations are notably resistant, whereas the Martinsburg, Romney, Greenbrier, and Canaan are especially weak. The Jennings formation is of intermediate resistance, with the Shenandoah and the Lewistown somewhat weaker and the Rockwood and Monterey somewhat more resistant. The relative resistance of these formations is not necessarily the same in other regions, but the method should have application elsewhere.

SUMMITS OF THE STAUNTON QUADRANGLE

Altimetric distribution and area.—The 533 summits of the Staunton quadrangle have a combined area of 21.75 square miles, which is about 2.3 per cent of the total area of the quadrangle. These summits range from 1,300 feet near Middle River in the east to 4,473 feet on Elliott Knob near the center of the quadrangle. Three prominent maxima are evident on the altimetric curve—the lowest at 1,500–1,700 feet, the second at 2,000 feet, and the highest at 3,000–3,100 feet. The maxima at 1,500–1,700 feet and at 2,000 feet represent summits in the Great Valley and in the Calfpasture and Little Calfpasture river valleys. Most of the 2,000-foot summits are along and near the divide between the Potomac and James drainage systems in the south-central part of the quadrangle around Summerdean, Middlebrook, and Moffett Creek. They are much more prominent on the James side than on the Potomac side of the watershed.

Three possible explanations are offered for the break between the 1,500–1,700 and the 2,000-foot maxima. (1) The 1,500–1,700-foot maximum represents the Harrisburg peneplain, and the 2,000-foot summits are low monadnocks on this peneplain underlain by cherty beds or other rock distinctly more resistant than the surrounding limestone. (2) Both maxima represent the Harrisburg peneplain, which slopes from 2,000 feet in the south to 1,500 feet in the northeastern part of the quadrangle, the curve being broken because of

pronounced local dissection caused by recent capture of some of the Potomac drainage by the James. (3) The 2,000-foot summits may represent a peneplain, higher than the Harrisburg, which has been almost completely destroyed except along the headwater drainage. This third proposal should be invoked only in case neither of the other two is satisfactory.

It is difficult to evaluate the importance of rock resistance in determining the location and altitude of the 2,000-foot summits. The rock is more or less cherty, as it is in practically all the low hills and ridges in the Great Valley. But the relative scarcity of summits at 1,800 and 1,900 feet and the flat tops of many of the 2,000-foot hills are strong arguments that rock resistance is not the sole explanation for the latter.

The second proposal is, I think, sufficient to account for the phenomenon. If those summits on the Potomac side of the divide only are considered, there is a gradual upstream rise to 2,000 feet at the divide. On the James side, however, the 2,000-foot upland, deeply trenched by streams, extends from the divide for a few miles, and there the surface drops off rapidly. There are almost no summits at 1,800 and 1,900 feet, these altitudes falling on the slopes of the steep-sided valleys that are cut into the surface. It is the scarcity of 1,800- and 1,900-foot summits on the James side of the divide that causes the pronounced re-entrant angle on the altimetric curve. Apparently the Potomac drainage formerly extended farther south in the Great Valley. Tributaries of the James, with steeper gradients than those of the Potomac, have tapped some of the northward-flowing drainage. The area thus transferred from the Potomac to the James is now trenched to a depth of 300–400 feet, and undissected portions remain as flattish hilltops.

If the foregoing observations are correct, peneplanation, separate from Harrisburg peneplanation, is not necessary to explain the sharp re-entrant angle on the altimetric curve between 1,700 and 2,000 feet. Had not piracy and its attendant dissection occurred, the curve would be continuously convex from 1,500 to 2,000 feet.

The maximum at 3,000–3,100 feet is approximately midway between the Harrisburg level and the elevation of the highest summits. This middle bulge is more sharply localized than the corresponding

bulge of the Monterey curve, a condition due, in part at least, to the summits of the Bullpasture Mountain, Walkers Mountain, and

TABLE 2

TABULATION OF SUMMITS ON THE FORMATIONS IN THE STAUNTON QUADRANGLE

Altitude of Summits (Feet)	Shenandoah Limestone (334.03 Sq. Mi.)	Martinsburg Shale (39.42 Sq. Mi.)	Tuscarora Sandstone (18.87 Sq. Mi.)	Rockwood Formation (20.74 Sq. Mi.)	Lewistown Limestone (48.82 Sq. Mi.)	Monterey Sandstone (26.16 Sq. Mi.)	Romney Shale (57.48 Sq. Mi.)	Jennings Formation (255.21 Sq. Mi.)	Hampshire Formation (110.83 Sq. Mi.)	Pocono Sandstone (29.99 Sq. Mi.)	Totals
4,400	.....	.....	.....	.....	.....	.....	.....	.....	1	2	3
4,300	.....	.....	.....	.....	.....	.....	.....	.....	.....	1	1
4,200	.....	.....	1	.....	.....	.....	.....	.....	6	.....	7
4,100	.....	.....	3	.....	.....	.....	.....	.....	3	3	9
4,000	.....	.....	1	.....	.....	.....	.....	.....	6	.....	7
3,900	.....	.....	.....	.....	.....	.....	.....	.....	2	1	3
3,800	.....	.....	.....	.....	.....	.....	.....	1	7	3	11
3,700	.....	.....	.....	.....	.....	.....	.....	.....	7	1	8
3,600	.....	.....	.....	.....	.....	.....	.....	1	4	1	6
3,500	.....	.....	.....	.....	1	.....	.....	.....	9	.....	10
3,400	.....	.....	.....	.....	.....	.....	.....	.....	9	3	12
3,300	.....	.....	1	2	2	1	.....	.....	6	.....	12
3,200	.....	.....	1	2	10	4	.....	2	2	.....	21
3,100	.....	.....	2	8	7	4	.....	3	4	.....	28
3,000	.....	.....	7	1	7	7	.....	6	2	3	33
2,900	.....	.....	3	2	8	4	.....	7	4	1	29
2,800	.....	.....	.....	3	.....	3	.....	9	1	1	17
2,700	.....	.....	2	1	3	1	.....	9	4	3	23
2,600	.....	.....	1	.....	1	2	.....	17	.....	.....	21
2,500	.....	.....	1	2	.....	1	.....	8	1	3	16
2,400	.....	.....	.....	.....	.....	1	.....	5	1	1	8
2,300	1	.....	.....	1	1	.....	1	12	1	1	18
2,200	.....	.....	.....	.....	.....	.....	1	8	.....	4	13
2,100	4	.....	.....	.....	.....	.....	1	6	.....	1	12
2,000	20	.....	1	2	1	.....	1	8	.....	.....	33
1,900	14	.....	2	1	.....	.....	3	6	.....	.....	26
1,800	13	.....	1	1	1	1	3	3	2	.....	25
1,700	26	.....	1	.....	.....	.....	2	8	1	.....	38
1,600	32	2	.....	.....	.....	.....	1	2	1	.....	38
1,500	25	2	.....	.....	.....	.....	.....	.....	.....	.....	27
1,400	11	4	.....	.....	.....	.....	.....	.....	.....	.....	15
1,300	4	.....	.....	.....	.....	.....	.....	.....	.....	.....	4
Total	150	8	28	26	42	29	13	121	83	33	.....
Grand total	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	533

Sideling Hill unbreached anticlines at 3,000–3,100 feet, which accentuate the curve at that altitude. This maximum is considered to be

the normal result of long-continued erosion on rocks of various degrees of resistance and is believed to have no cyclic significance. No maximum above that at 3,000–3,100 feet is prominent enough to suggest planation.

Formations on which summits occur.—Table 2 shows the area in square miles of each formation in the Staunton quadrangle and the number and altitudinal distribution of summits on each formation. From the table it is obvious that the summits at high altitude are limited to a few formations, those at very low altitude are also limited to a few formations, and those at intermediate altitudes are found on most of the formations. Summits above 3,800 feet occur only on the synclinal Pocono and Hampshire formations in the Shenandoah and Great North Mountains and the anticlinal Tuscarora (Massanutten) in Jack Mountain. Summits below 1,600 feet occur only on the Shenandoah limestone and the Martinsburg shale in the Great Valley. The eight summits on the Martinsburg shale average lower in altitude than the summits on the Shenandoah limestone. The Hampshire formation, probably the most variable in lithology of the formations in this quadrangle, has summits distributed through the greatest range of altitude. The number of summits on most of the formations is not in proportion to their areas of outcrop. Thus, some of the formations are prominent summit-makers while others have relatively few summits.

SUMMITS OF THE HARRISONBURG QUADRANGLE

Altimetric distribution and areas.—The 352 summits in the Harrisonburg quadrangle comprise about 1½ per cent of the total area of the quadrangle. The summits range in altitude from 600 feet in the Piedmont to 3,600 feet in the Blue Ridge. All of the summits below 1,100 feet are in the Piedmont. The portion of the Piedmont included within the quadrangle is too small to warrant a statement of the general altitude of summits in the Piedmont province. However, their low altitudes, as compared to the valley west of the Blue Ridge, evidently result from nearness to the James, whereas the drainage in the Great Valley is far distant from the Potomac. The great maximum at 1,300–1,500 feet is the Harrisburg peneplain level in the Great Valley. This surface slopes northeastward with dimin-

ishing slope downstream from 2,000 feet on the divide in the southern part of the Staunton quadrangle to about 1,400 feet in the north-

TABLE 3

TABULATION OF SUMMITS ON THE FORMATIONS IN THE HARRISONBURG QUADRANGLE

Altitude of Summits (Feet)	Pre-Cambrian Granite, Gneiss, and Gabbro (236.24 Sq. Mi.)	Catoctin Greenstone (112.62 Sq. Mi)	Lower Cambrian Sandstone and Shale (102.77 Sq. Mi.)	Shenandoah Limestone (352.41 Sq. Mi)	Martinsburg Shale (117.51 Sq. Mi)	Tuscarora Sandstone (13.84 Sq. Mi.)	Oriskany-Hamilton (3.12 Sq. Mi.)	Totals
3,600	.....	.....	1	.....	.....	.....	.....	1
3,500	.....	.....	.....	.....	.....	.....	.....	.....
3,400	1	.....	.....	.....	.....	.....	.....	1
3,300	.....	3	2	.....	.....	2	.....	7
3,200	1	.....	2	.....	.....	.....	.....	3
3,100	.....	3	2	.....	.....	.....	.....	5
3,000	.....	7	.....	.....	.....	1	.....	8
2,900	.....	2	2	.....	.....	3	.....	7
2,800	1	3	10	.....	.....	2	.....	16
2,700	1	4	2	.....	.....	.....	.....	7
2,600	3	4	3	.....	.....	3	.....	13
2,500	1	3	7	.....	.....	2	.....	13
2,400	.....	.....	11	.....	.....	.....	.....	11
2,300	2	2	5	.....	.....	.....	.....	9
2,200	.....	3	7	.....	1	.....	.....	11
2,100	.....	3	8	.....	1	.....	.....	12
2,000	.....	1	7	.....	.....	.....	.....	8
1,900	2	1	1	1	.....	.....	.....	5
1,800	1	.....	5	1	.....	.....	.....	7
1,700	2	2	12	3	1	.....	.....	20
1,600	4	.....	4	9	.....	.....	.....	17
1,500	3	.....	3	21	.....	.....	.....	27
1,400	.....	.....	1	53	7	.....	.....	61
1,300	8	.....	.....	17	18	.....	.....	43
1,200	3	.....	.....	1	3	.....	.....	7
1,100	3	2	1	1	.....	.....	.....	7
1,000	11	2	1	.....	.....	.....	.....	14
900	3	.....	.....	.....	.....	.....	.....	3
800	6	.....	1	.....	.....	.....	.....	7
700	1	.....	.....	.....	.....	.....	.....	1
600	1	.....	.....	.....	.....	.....	.....	1
Total	58	45	98	107	31	13	.....	.....
Grand total	.....	.....	.....	.....	.....	.....	.....	352

ern part of the Harrisonburg quadrangle. In addition to the general northeast slope, the surface slopes from both sides toward the main

streams. Higher summits are too few, small, and discordant to suggest a peneplain above the Harrisburg. The absence of the pronounced middle bulge on the curve, so prominent on the curves of the Staunton and Monterey quadrangles, is probably due to the scarcity of rocks of intermediate resistance and to a more advanced stage of erosion. Between the resistant crystallines and Lower Cambrian sandstones of the Blue Ridge, on the one hand, and the weak Shenandoah limestone and Martinsburg shale of the Great Valley, on the other, there is no great amount of moderately resistant rock, such as the Jennings and Hampshire formations farther west. The advanced stage of erosion results from the broad, uninterrupted outcrop of weak rock and the downstream position of the region.

Formations on which summits occur.—Pre-Cambrian granite, gneiss, and gabbro, the Catoctin greenstone, and the Lower Cambrian sandstones and shale comprise the Blue Ridge and the Piedmont. The Shenandoah limestone and Martinsburg shale in the Great Valley underlie about half of the quadrangle. The Tuscarora, Oriskany, and Hamilton are in the synclinal Massanutten Mountain. In proportion to area, the Tuscarora and Lower Cambrian sandstones are especially prominent summit-makers. It should be noted that summits on the Martinsburg shale average lower than those on the Shenandoah limestone (see Table 3).

SUMMITS ON LARGE-SCALE MAPS

Fearing that the scale of the Monterey, Staunton, and Harrisonburg sheets might be too small and the contour interval too large to reveal accurately significant summit maxima, altimetric curves of the summits were constructed from the larger maps, which cover nearly half of the region. Two of these curves are shown in Figure 3.[11]

The curves constructed from the large-scale maps conform closely

[11] The Cass, West Virginia–Virginia, and the Warm Springs, Virginia–West Virginia (only the West Virginia portion is mapped), 15-minute quadrangles with 50-foot contour interval and the Mountain Grove, Virginia–West Virginia (only the Virginia portion is mapped), and Warm Springs Run, Virginia, 7½-minute quadrangles with 20-foot contour interval cover nearly all of the western half of the Monterey quadrangle. The Waynesboro, Virginia, and Elkton, Virginia, 15-minute quadrangles, each with a contour interval of 20 feet in the lowlands and 50 feet on the ridges, cover the southwestern and northeastern quarters of the Harrisonburg quadrangle.

with the curves of the same areas made from the smaller-scale maps. Because of the smaller contour interval the former are more ragged, but the main maxima and minima are essentially the same. The smaller the area included in a curve the more prominently do local

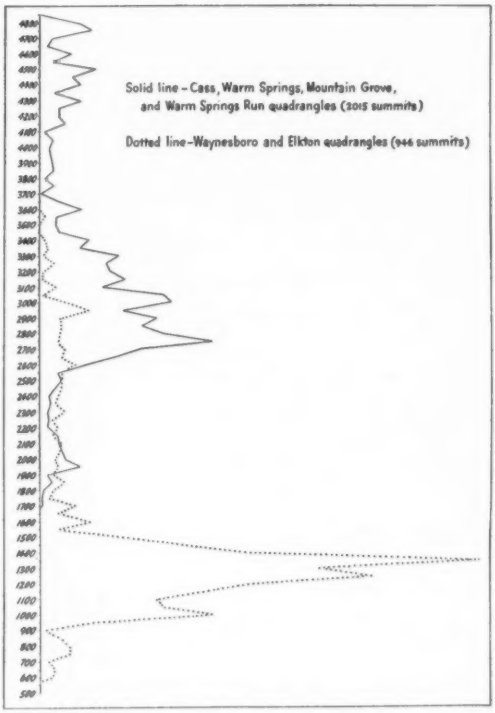
Solid line – Cass, Warm Springs, Mountain Grove, and Warm Springs Run quadrangles (2015 summits)
Dotted line – Waynesboro and Elkton quadrangles (946 summits)

FIG. 3.—Altimetric curves of the Cass, Warm Springs, Mountain Grove, and Warm Springs Run quadrangles and of the Waynesboro and Elkton quadrangles.

groups of summits appear. Thus, the summits on the Allegheny Plateau at about 4,750 feet, those on Allegheny Mountain at about 4,500 feet, and those on the Blue Ridge at about 2,950 feet are more prominent in Figure 3 than they are in Figure 2.

CONCLUSIONS AS TO SUMMITS

The summits, especially the upland summits, are small in area. Less than 2 per cent of the total area of the Monterey, Staunton, and Harrisonburg quadrangles is included within the summit contours. When plotted on a scale, the summits do not appear so nearly accordant as when seen from a commanding elevation in the field.

By far the most prominent erosion plane in this region is the Harrisburg. This surface is best developed in the Harrisonburg quadrangle west of the Blue Ridge, where the great preponderance of summit areas is at about 1,400 feet. It is well developed in the eastern part of the Staunton quadrangle at 1,500–2,000 feet. In the Monterey quadrangle the position of the Harrisburg peneplain is scarcely recognizable on the summit curve. The valleys are fairly narrow, with few summits in them and rise upstream rather rapidly. In all three quadrangles the Harrisburg surface rises upstream and away from the main streams, the rise becoming steeper toward the headwaters.

Recent capture of some of the headwaters of the Potomac by tributaries of the James is believed to account for the relative scarcity of 1,800- and 1,900-foot summits in the southern part of the Staunton quadrangle, a scarcity which causes the 2,000-foot level to appear falsely as separate from the Harrisburg level.

It is believed that the summit maximum at about 3,000 feet in the Monterey and Staunton quadrangles has no cyclic significance but is the normal result of noncyclic erosion on rocks of various degrees of resistance. In case one does assign cyclic significance to this summit maximum, the peneplain it represents slopes westward, for it is slightly higher in the Staunton quadrangle than it is in the Monterey quadrangle. A peneplain at that level could not have been nearly perfect, as the higher ridges rise far above it.

Summits above the middle maximum are so few in number, small in area, and discordant in altitude that I should not like to fix the position of any peneplain on the basis of the evidence they furnish. If one or more higher peneplain once existed in this region, they must have been so completely destroyed as to be unrecognizable on the charts of the present summits.

RIDGES

GENERAL STATEMENT

Table 4 is a summary of pertinent data on the principal ridges from the Allegheny Front to the Blue Ridge between latitudes 38° N. and 38°30′ N. This area extends somewhat beyond the boundary of the Monterey quadrangle on the southwest and slightly beyond the boundary of the Harrisonburg quadrangle on the northeast. Ridges included in the table are those that are continuous within the region for 10 miles or more and rise to a height of 1,000 feet or more above the adjacent valleys. Certain relationships between the data in Table 4 seem significant. The ridges are listed in order from west to east. The average altitudes of the summits along the crest lines (col. 2) give the elevations of the ridges as they would be without notches. A fair average altitude of the whole crest line may be obtained by averaging the altitudes of the notches (col. 4) with those of the summits. Slopes of the ridge sides (col. 7) were determined according to the method explained by A. B. Cozzens.[12]

ALTITUDES OF RIDGES

From the Allegheny Front eastward there is in general a decline in the altitudes of the ridges. This decline is by no means regular or continuous; lower ridges alternate with higher ones. Obviously, not all of the ridge crests can be remnants of an eastward-sloping peneplain. Nor can they have been uniformly reduced from such a peneplain. If the valleys were filled to the tops of the ridges, the surface would still be corrugated with considerable relief. By considering only certain of the higher ridges—Allegheny Front, Allegheny Mountain, Jack Mountain, Warm Springs Mountain, Shenandoah Mountain, and Blue Ridge—a fairly gradual and uniform eastward slope is discernible. If these higher ridges represent an eastward-sloping peneplain, the lower, intervening ridges have been variously reduced from that level.

It seems superfluous to consider an upland or summit peneplain if the various altitudes of the ridges can otherwise be satisfactorily explained. The following factors are believed to account sufficiently for the various altitudes of the ridges: (1) different rock formations,

[12] "An Angle of Slope Scale," *Jour. Geomorph.*, Vol. III (1940), pp. 52–56.

TABLE 4

TABULATION OF DATA ON THE RIDGES FROM THE ALLEGHENY FRONT TO THE BLUE RIDGE, BETWEEN LATITUDES 38° AND 38° 30′ N.

Name of Ridge	Average Altitude of Summits (Feet)	Highest Summit (Feet)	Average Altitude of Gaps (Feet)	Altitude of Lowest Gap (Feet)	Average Distance Between Gaps (Miles)	Slope of Sides (Per Cent)	Average Height Above Valleys (Feet)	Principal Formations	Structure
Allegheny Front*	4,293	4,800	3,796	3,200 (dry)	2	37	2,043	Blackwater	Nearly flat syncline
Marlin Mountain–Sandy Ridge	3,218	3,400	2,828	2,200 (water) 2,600 (dry)	1	20	837	Jennings	Monocline
Browns Ridge–Michael Mountain	3,456	3,700	2,970	2,250 (water) 2,800 (dry)	2.9	32.5	910	Lewistown Tuscarora	Anticline
Allegheny Mountain†	4,022	4,600	3,605	2,500 (dry)	1.77	30	1,482	Pocono Hampshire Jennings	Syncline
Little Mountain	3,375	4,000	3,063	2,200 (water) 2,500 (dry)	2	39	900	Jennings Tuscarora	Monocline
Back Creek–Monterey Mountain	3,679	4,100	3,145	1,700 (water) 2,500 (dry)	3	37	1,254	Tuscarora	Anticline-monocline
Jack Mountain	3,985	4,400	3,546	3,100 (dry)	2	43.5	1,648	Tuscarora	Anticline
Warm Springs Mountain	3,800	3,800	2,900	2,900 (dry)	One only	41.5	1,685	Tuscarora	Anticline
Tower Hill–Bullpasture Mountain	3,046	3,300	2,645	1,700 (water) 2,100 (dry)	1.18	33	978	Monterey Lewistown Tuscarora	Anticline
Shaws Ridge	2,940	3,200	2,600	2,500 (dry)	2.8	28	744	Jennings	Monocline
Shenandoah Mountain	3,792	4,410	3,350	2,700 (dry)	1.32	41	1,900	Pocono Hampshire	Syncline
Walkers Mountain	3,100	3,100	None	None	None	37	1,322	Tuscarora	Anticline
Great North–Narrowback Mountain	2,855	4,473	2,380	1,600 (water) 1,800 (dry)	2.12	44	1,054	Pocono Hampshire	Syncline-monocline
Little North Mountain	2,811	3,200	2,425	1,800 (water) 2,300 (dry)	1.88	42	1,061	Tuscarora	Monocline
Blue Ridge	3,130	3,800	2,567	1,800 (dry)	2.14	35	2,100	Catoctin	Anticline

* Back Allegheny Mountains on Monterey sheet.

† Allegheny Front on Monterey sheet.

(2) structure, (3) width of outcrop of ridge-makers, (4) altitude of anticlinal folds, and (5) nearness to main streams. The Tuscarora, Hampshire, Pocono, and Blackwater formations make the highest ridges. As to structure, synclinal ridges are highest, anticlines are next, and monoclines are on the average lowest. Several of the monoclinal ridges consist of Tuscarora sandstone. Those Tuscarora ridges where the dip is gentle and the outcrop is broad are higher than the ridges of the same formation where the dip is steep and the outcrop is narrow. The altitudes of some ridges—Back Creek Mountain, south of Fodder House, Jack Mountain, north and south of Bolar Valley, Warm Springs Mountain, north of Boner Mountain, and Walkers Mountain—depend on the altitude of anticlinal folds of the Tuscarora sandstone and decline in elevation with the pitch of these folds. The two mountains of greatest altitude are important drainage divides. The Allegheny Front in the northwestern part of the Monterey quadrangle is the divide between the Monongahela and the New River drainage and is the highest ground in this region. Southward this mountain declines in altitude as it becomes the divide between two branches of the New River system. Allegheny Mountain, the second highest mountain in this region, is the drainage divide between the Atlantic and the Gulf. In the northern part of the Monterey quadrangle, where this mountain is highest, runoff goes to the Potomac, to the James, and to the Greenbrier. Southward, downstream between the James and the Greenbrier, the altitude declines.

EVENNESS OF CREST LINES

The spacing and depth of notches in the ridges should give a fairly accurate picture of the evenness of crest lines. Only those gaps more than 100 feet deep are counted here. There seems to be no relation between altitude of the ridges and the spacing of notches. Some of the highest, as well as some of the lowest, ridges are very knobby; and some of the highest, as well as some of the lowest, have very uniform crest lines. Although it is difficult to formulate a rule regarding evenness of crest lines, it appears that structure and the variety of rocks are important. Most of the anticlines are comparatively even-crested, especially if composed of an unbreached resistant formation. Bullpasture Mountain is an exception to the gen-

eral rule because of the variety of rocks along its crest—the Tuscarora, Lewistown, and Monterey formations each comprising some of the summits. The Monterey formation is especially inclined to knobby summits.

In the southeastern part of the Monterey quadrangle and the southwestern part of the Staunton quadrangle is a series of anticlinal ridges with even crestlines. Walkers Mountain has an even crest at 3,100 feet for a distance of 6 miles. As pointed out by Wright,[13] this crest is formed by a denuded anticline of the Tuscarora sandstone and is not related to any peneplain surface. Six miles to the southeast of Walkers Mountain another denuded arch of the Tuscarora forms the even crest of Black Oak Ridge at 2,000 feet. To the south of Walkers Mountain the even crest of Mill Mountain at 2,500 feet is formed by a similar anticlinal fold of the Tuscarora sandstone. The monoclinal Monterey Mountain, west of Monterey, has some long summit stretches at 4,000 feet; Jack Mountain, west of Bolar Valley, has a long, narrow summit at 3,400 feet. The writer agrees with Rich[14] that such summits are the natural result of structure and are not dependent on peneplanation.

Average depth of the gaps, obtained by subtracting the figures in column 4 from those in column 2 in the table, is not proportional to the altitudes of the ridges. In general, the higher a ridge is above its surroundings (col. 8) the deeper the gaps, though there are exceptions to this rule. A low ridge cannot have deep gaps, even though breached to its base, whereas a high ridge may be deeply notched by gaps above its base line.

STEEPNESS OF SLOPES

Figures in Column 7 give the average percentage of slope of the ridge sides as measured from the crest down 800 feet on both sides at intervals of about 1 mile. As a rule, the higher ridges are steeper than the lower ones, though there are several exceptions. All the steepest ridges are composed of Tuscarora sandstone or have synclinal structure. Synclines of the Tuscarora formation do not occur in this region, except in Massanutten Mountain, which is not included in the table. Monoclines of the Tuscarora are steeper than the anti-

[13] "Physiography of the Upper James River . . . . ," *op. cit.*, p. 23.

[14] *Op. cit.*, p. 1701.

clines of that formation. For example, Monterey Mountain and the northern part of Little Mountain, monoclinal limbs of the breached Back Creek anticline, have slopes of about 40 per cent, whereas the unbreached southern portion of this anticline has a slope of about 30 per cent. Also, the limbs of the breached portion of Jack Mountain, on each side of Bolar Valley, are steeper than the unbreached portions of that anticline. Ridges of the Jennings formation have the gentlest slopes.

BREACHED ANTICLINES

In the Monterey quadrangle are three anticlines of the Tuscarora sandstone which have been breached, and canoe-shaped valleys have been excavated between the limbs. North of Fodder House, in the center of the quadrangle, the breached Back Creek anticline extends as Little Mountain on the west and Monterey Mountain on the east. South of Boner Mountain, in the southern part of the quadrangle, the Warm Springs anticline is breached. The Jack Mountain anticline, in the eastern part of the quadrangle, is breached between Duncan Knob on the south and Sounding Knob on the north. In each of these anticlines, as in most anticlines in this part of the Appalachians, the western limb has the steeper dip, and therefore, narrower outcrop of the ridge-making formation. In each case the eastern ridge is higher than the western. In the Back Creek anticline the average difference in altitude of the two ridges is 472 feet; in Warm Springs Mountain and also in Jack Mountain the difference is about 300 feet between the eastern and the western ridges.

The amounts of differential lowering of these paired ridges of the breached anticlines correspond approximately with the altitudes at which the restored folds of the Tuscarora were breached. All the three folds appear to have about the same degree of symmetry. Restoration of the folds on the basis of measured dips of the strata indicates that the Tuscarora sandstone in the Warm Springs and Jack Mountain anticlines was breached at about 5,500 feet, and in the higher Back Creek anticline at about 7,000–7,500 feet. These altitudes at which the anticlines were probably breached are above the level at which anyone has placed an upland peneplain in this region. Were the folds all truncated by a peneplain at a common level, the amount of subsequent differential lowering of the paired

limbs should be about the same for each anticline. Instead, however, the amounts of differential lowering are proportionate to the calculated altitudes at which the anticlines were breached. This fact suggests that denudation of the ridges has been continuous since the anticlines were breached.

There are, of course, several factors that take part in the differential reduction of ridges. However, it is believed that examination of many cases of differential lowering and its relation to structure in various localities should be of importance in indicating either the presence or the absence of an upland peneplain; also a ratio between thickness of outcrop and rate of reduction might be established.

FLAT TOPS

In the northwestern part of the Monterey quadrangle the Appalachian Plateau might be considered an upland flat, although the relief is nearly a thousand feet. The structure is a very broad syncline with the resistant Blackwater (Pottsville) formation at the surface except where Cheat River, following the axis of the syncline, has cut through to the underlying Canaan (Mauch Chunk). From the eastern edge of the plateau (Allegheny Front), where the highest knobs rise above 4,800 feet, the surface drops down the steep eastward-facing escarpment to an intermediate bench on the Pocono or Hampshire sandstone at about 3,000 feet altitude. From the intermediate bench a second escarpment about 500 feet high leads down to the Greenbrier River. Thus, there are three topographic levels—the plateau upland at about 4,500 feet, the intermediate bench at about 3,000 feet, and the Greenbrier Valley at about 2,500 feet.

Considering only the surface, these three levels suggest three different baselevels. But, when the structure is considered, their cyclic significance is doubtful. The plateau surface conforms approximately with the altitude of the resistant Blackwater formation, more or less trenched by Cheat River and its tributaries. It is not known whether any younger rock ever covered the Blackwater in this region. If so, it has been stripped off. The intermediate bench is underlain by nearly flat-lying sandstone, from which the weak Greenbrier limestone and Canaan shale have been more or less stripped. The present level of the Greenbrier River is that of a graded stream, which may be slowly lowered further without uplift. The topog-

raphy here is somewhat analagous to that of the steplike walls of a canyon where weak and resistant rocks alternate. Multiple cycles do not seem necessary to explain such features.

Top of Allegheny, on Allegheny Mountain, in the northern part of the Monterey quadrangle, is an upland with a flattish surface, the highest parts of which rise to more than 4,600 feet. This mountain top is dissected by small streams to a depth of about 800 feet. The upland surface of this mountain top is formed by resistant Hampshire or Pocono sandstone in the trough of a broad syncline. Because the upturned beds on both sides of the synclinal axis are truncated, this surface has been considered as a remnant of the Schooley[15] peneplain. Such truncation is a necessary consequence of peneplanation. It would seem, also, that truncation might result without peneplanation, because the axis of a syncline, unless drained by a longitudinal stream, can be lowered only as the upturned beds on the sides are worn down.

HYPSOGRAPHIC CURVES

GENERAL STATEMENT

The hypsographic curve, in which altitude is plotted against area, has long been used to represent the generalized major relief features of the earth. This curve should be effective in more detailed work and has been so used in England. According to Woolridge and Morgan:

> The ideal curve for an area which had suffered uninterrupted denudation by water-erosion, after a single simple uplift, would be regular in form, concave upwards, representing as it were, the generalized curve of water-erosion. The preservation of portions of planation surfaces or "platforms" must modify the curve, by superimposing a convexity on its simple concave form, at points corresponding with the general heights of the several platforms.[16]

The data for the hypsographic curves of the Monterey, Staunton, and Harrisonburg quadrangles (Fig. 4) were obtained by measuring with a planimeter the area enclosed by each successive contour line and expressing each area as a percentage of the total area of the quadrangle.

[15] Wright, "Physiography of the Upper James River . . . . ," *op. cit.*, pp. 13–14.

[16] *Op. cit.*, p. 261.

MONTEREY QUADRANGLE

Above 3,200 feet there is no convexity or flattening of the hypsographic curve of the Monterey quadrangle to suggest planation. About 20 per cent of the surface of the quadrangle is above 3,200 feet. From 3,200 feet down to 2,500 feet the curve flattens gradually. About 44 per cent of the surface lies between 2,500 feet and 3,200 feet. Below 2,500 feet the curve steepens gradually, becoming

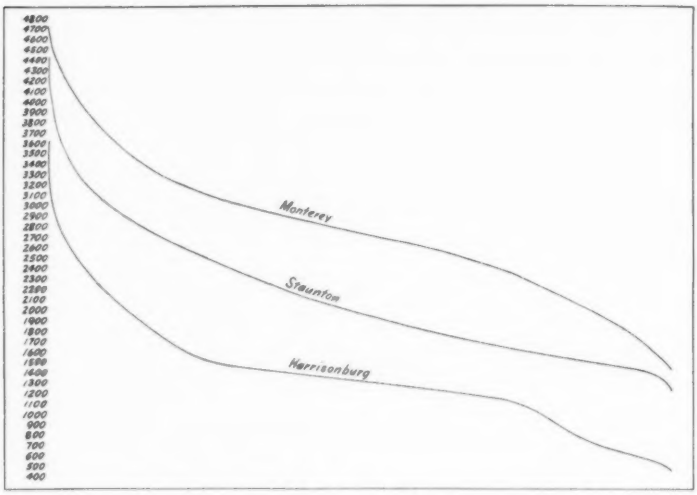

FIG. 4.—Hypsographic curves of the Monterey, Staunton, and Harrisonburg quadrangles.

marked below 1,800 feet. About 27 per cent of the surface is between 1,800 and 2,500 feet altitude. That part of the curve below 3,200 feet includes the valleys, the highest of which reaches 3,200 feet on the Potomac-James divide, some of the low ridges, and the bench on the Allegheny Front. Some degree of planation, presumably Harrisburg, followed by rejuvenation, is here indicated. Planation was practically limited to the weak rocks of the valleys and even there was not perfect. The valleys are probably more accurately designated as mature valleys rather than parts of a peneplain. It is noticed that the flattening of the curve is not sharply localized but extends through quite a wide range of altitude. One cause for

this is the fact that the valleys slope downstream at an average of about 30 feet per mile; another reason is found in the different elevations of the different valleys. The elevation of any valley, as well as the amount of its downstream slope, seems to depend largely on the size of its stream. Difference in the amounts of response to rejuvenation might, to some extent, explain the different altitudes of the various valleys, but it may well be doubted that they were all at the same level before rejuvenation.

STAUNTON QUADRANGLE

The hypsographic curve of the Staunton quadrangle does not indicate peneplanation at any upland or intermediate level. From 1,900 feet to 1,400 feet the gradual flattening of the curve, so as to include more than 42 per cent of the surface of the quadrangle, shows the Valley (Harrisburg) peneplain. This peneplain is well developed in the Great Valley; it is fairly well developed in Calfpasture River valley between Great North and Shenandoah mountains. Farther west the narrow valleys rise in elevation and reach a maximum of 2,800 feet at the head of Bullpasture River on the divide between the James and the Potomac. Steepening of the curve below 1,400 feet shows post-Harrisburg trenching of the Valley floor.

HARRISONBURG QUADRANGLE

A very pronounced platform occurs at 1,100–1,500 feet, which includes nearly one-half of the quadrangle. This platform represents the Harrisburg level in the Great Valley. Sharply separated from the Great Valley level is a second platform at 500–700 feet, which shows the Piedmont level in the southeastern part of the quadrangle. According to Wright,[17] the Valley level articulates with the Piedmont level through the James and Potomac gaps in the Blue Ridge, and both belong to the Harrisburg cycle. In the Harrisonburg quadrangle the two surfaces differ in altitude because of difference in distance to the main streams. The Piedmont section is only a short distance from the James, whereas the Valley section is quite far from the Potomac. The relationship between these two regions illustrates the danger of using the criterion of altitude alone in correlating peneplanes. Above the Valley level the curve does not suggest any peneplain level.

[17] "The Newer Appalachians of the South, ... ," *op. cit.*, p. 19.

DRAINAGE

Shavers Fork of Cheat River drains about 25 square miles of the northwestern part of the Monterey quadrangle northward to the Monongahela River. About 277 square miles of the western part of the Monterey quadrangle are drained southward to the New River by the Greenbrier and its tributaries. The rest of the area of the three quadrangles is about equally divided between tributaries of the James and those of the Potomac rivers, the Potomac drainage going northward and the James drainage going southward and eastward. Nearly 90 per cent of the region is drained to the Atlantic. The main drainage divide, between the Atlantic and the Mississippi, follows the crest of Allegheny Mountain, about 12 miles to the east of the Allegheny Front.

In an earlier paper the writer[18] proposed that all of the drainage west of the Blue Ridge to the main divide between the east and the west is captured drainage. A few facts may here be pointed out which seem to support that interpretation.

As a rule, the drainage is adjusted to structure. The streams are mostly longitudinal, following the weak-rock outcrops. Because of the regional slope and the pitching and irregular folds, however, it would be impossible to drain all of the weak-rock areas without some streams crossing resistant rocks. Most of the water gaps are in monoclinal ridges; four cut through anticlines. There are no water gaps in the synclinal mountains.

Although the streams on both sides of the main drainage divide are fairly well adjusted to structure, adjustment on the east is apparently better than it is on the west of the divide. In proportion to area, water gaps are more numerous on the west than on the eastern side of the divide. Tributaries of Greenbrier River flow westward from the divide on Allegheny Mountain across the structure. North Fork, Suttleton Creek, Thorny Creek, Knapp Creek, and others to the south cut through the Marlin Mountain–Sandy Ridge monoclinal range. Suttleton Creek and Knapp Creek also cross the Browns–Michael Mountain anticline, thus cutting directly through two ridges in succession. Suttleton Creek crosses the Browns–Michael Mountain arch near the northern end of this northward-pitch-

[18] "Drainage Evolution in the Southern Appalachians," *Bull. Geol. Soc. Amer.*, Vol. L (1939), pp. 1323–56.

ing fold and was apparently superposed onto the hard rock from the overlying Lewistown limestone. Knapp Creek crosses the anticline at about the central, or highest, part of the two-way pitching fold. This creek, also, was evidently superposed onto the resistant sandstones from the overlying Lewistown limestone and has persisted in its course until it has cut deeply into the resistant rock. Of all the streams in this region, Knapp Creek, where it crosses the structure, appears to be most out of adjustment.

East of the drainage divide are two water gaps in anticlinal mountains. Back Creek crosses the Back Creek anticline near the southern end of this southward-pitching fold, and Bullpasture River crosses the Tower Hill–Bullpasture Mountain anticline at a sag in the structure. In each of these cases the stream crosses the anticline where the structure is relatively low and has cut through the limestone into the underlying sandstone. At no place east of the drainage divide, in this region, does one find a transverse stream cutting directly through more than one ridge.

Formation of gaps by headward erosion is a corollary of the hypothesis of the westward shifting of the main drainage divide. The divide is now on Allegheny Mountain. If the original divide had the same location, one must admit that gaps of considerable depth can be formed without the aid of through-flowing streams. Between Warwick Run and Knapp Creek a dry gap in Allegheny Mountain is about 1,200 feet deep. Six miles to the north in the same mountain Galfred Gap is about 800 feet deep. There are many others of lesser depth. If the original divide were not on Allegheny Mountain, it has migrated either eastward or westward. Either alternative would necessitate the reversal of drainage.

CONCLUSION

Interpretation of geomorphic history is largely a matter of the consideration of the balance of probabilities; conclusions must change as new facts become available or new meaning is given to old facts. Probably we should have detailed tabulation of data, quadrangle by quadrangle, for much of the Appalachian Mountains before we can feel confident of our interpretation of the topographic history of that region.

On the basis of the data at hand certain tentative conclusions re-

garding the topographic history of the Monterey-Staunton-Harrisonburg region may be drawn. My interpretation of the data supports Wright[19] on the importance of Harrisburg erosion, the differential lowering of ridges, and the uncertainty of locating the Schooley peneplain; it agrees with Ashley[20] on the necessity for detailed knowledge of bedrock in drawing conclusions on physiographic history; it favors Rich's[21] conclusions that several of the criteria, such as regional truncation of deformed strata, even-crested ridges, and horizontal skyline—commonly used as evidences of peneplanation—are unreliable. In fact, the writer agrees in general with the above-named authors on most of their conclusions in the papers cited.

However, one important element, the Schooley, or Upland, peneplain, commonly emphasized by writers on Appalachian topography, is not substantiated by the evidence in this report. The following four alternatives are suggested as reasons why, according to this paper, the Schooley peneplain is not apparent in the Monterey-Staunton-Harrisonburg region.

1. The methods used are not sufficiently precise. Many of the data were compiled from the 30-minute maps with the 100-foot contour interval, larger-scale maps not being available for part of this region. However, the data for parts of the region were checked on the 15-minute and $7\frac{1}{2}$-minute maps and were found to conform closely with the results obtained from the smaller-scale maps.

2. Evidence of the Schooley peneplain may be present, and the writer fails to recognize it. For example, the middle bulges on the altimetric curves of the Monterey and Staunton quadrangles, or the relatively flat tops of Cheat Mountain and Allegheny Mountain—features which the writer explained without recourse to peneplanation—may possibly have cyclic significance.

3. The Schooley peneplain, if it existed in this region, has been destroyed beyond recognition by subsequent erosion. Probably few geologists now believe that actual remnants of the Schooley peneplain are to be found in the present surface. Wright[22] says of the Ridge and Valley province: "All of the ridges have been reduced somewhat since the uplift of the Schooley peneplain and some more

[19] "The Newer Appalachians of the South . . . . ," *op. cit.*, pp. 14–15 and 18.

[20] *Op. cit.*, p. 1398. [21] *Op. cit.*, p. 1702.

[22] "The Newer Appalachians of the South . . . . ," *op. cit.*, p. 14.

than others." Ashley,[23] in his conclusion on the rate of erosion in Pennsylvania, says: "The present surface, though reflecting a single old peneplain, has been lowered not less than 100 feet on the hardest rocks and several hundred feet on softer rocks for each million years since the beginning of uplift." With such a great amount of differential lowering as Ashley suggests, which seems entirely reasonable, it would be practically impossible to locate accurately the Schooley peneplain, even it it were originally perfect.

4. The Schooley peneplain may never have developed in this region. I know of no reliable basis on which to decide between alternatives 3 and 4. Characteristics of the present topography might be the same in either case. Even by those who consider such features as even crests and breaks in slope unreliable criteria, general peneplanation of the Appalachian Highlands has seemed necessary to explain (*a*) the regional truncation of strata and (*b*) the supposed regional superposition of drainage across the structure. It is also argued that the region could not well have escaped peneplanation in the long time since the original uplift. As pointed out by Rich[24] and others, regional truncation of strata, though a necessary consequence of peneplanation, is also a necessary result of long-continued denudation without peneplanation. A hypothesis of drainage development, without recourse to regional peneplanation or regional superposition, has been discussed in an earlier paper by the writer.[25] Of course, the time since the original uplift of this region has certainly been long enough for one or more general planations; it has also been long enough for the destruction of such peneplains, if formed. There is also the possibility of many successive uplifts at intervals too brief for general planation. The Harrisburg cycle is an example of limited, or weak-rock, planation; possibly earlier peneplains in this region were also limited to weak rocks.

ACKNOWLEDGMENTS.—The writer wishes to acknowledge the helpful suggestions made by Dr. George H. Ashley, Miss Anastasia Van Burkalow, and Professor O. D. von Engeln. As some of the methods here employed have not been proved by long usage and the conclusions are therefore tentative, it is hoped that further criticisms will be proposed.

[23] *Op. cit.*, p. 1398.

[24] *Op. cit.*, p. 1698.

[25] "Drainage Evolution in the Southern Appalachians," *op. cit.*, pp. 1323–56.

CLASTIC CREVICE FILLINGS IN THE KEWEENAWAN LAVAS[1]

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ABSTRACT

The crevice fillings on the north shore of Lake Superior are irregular in their surface pattern, and the crevices probably resulted from thermal contraction in basaltic flows. The material was derived from erosion of the rocks exposed in the area, mostly basalt and basalt tuffs; but partly from associated felsites. The fillings were washed into the cracks and are older than the flow lying above the filling.

INTRODUCTION

Sandy and shaly fillings of crevices in the Keweenawan lavas of the Lake Superior region have been noted in the literature but have never been studied in much detail. A large number of clastic dikes have been found in other regions which result from the intrusion into joints and fissures of plastic material whose source was below the intruded beds. Consequently it was thought advisable to investigate the origin and nature of the crevice fillings in the Lake Superior lavas.

One of the earliest references to these clastic fissure fillings is that of N. H. Winchell,[2] who mentions the occurrence of brown, aluminous, or sandy layers in the lava flows. R. D. Irving[3] in his classic work on the Lake Superior region observed, "where the overlying sandstone bed has been removed, that large sections of the underlying diabase, sometimes many hundreds of feet in length, present the singular appearance of being intersected by veins of sandstone . . . . crossing each other, zig-zagging, and branching, like true vein-formed material." M. E. Wadsworth[4] introduced the term "claso-

[1] Published with the permission of the director of the Minnesota State Geological Survey.

[2] Minnesota Geol. Surv., *Ninth Annual Report* (1880), p. 42.

[3] "Copper-bearing Rocks of Lake Superior," *U.S. Geol. Surv., Mono. V* (1883), p. 140.

[4] "Sketch of the Iron, Gold, and Copper Districts of Michigan," *Mich. State Board Geol. Surv. Rept. for 1891–92* (1893), p. 147.

FIG. 1.—Index map

FIG. 2.—Crevice filling showing bedding especially near compass; zeolitization also may be seen.

lite" for "the various sedimentary deposits of sandstone and detrital that fill fissures." A. C. Lane[5] in his work on the Michigan flows said: "In fact this shaly or sandy sediment may work far down into the old lava beds or traps, following the cracks that formed as it cooled." A. E. Sandberg[6] mentions clastic stringers and uses them as one of his criteria for the distinction of separate flows, since they are found only at or near the tops of the flows. F. F. Grout and G. M. Schwartz[7] in the latest publication on the region mention clastic dikes in Lake County.

Fragmental fillings of crevices crop out along the shore of Lake Superior at many places from Duluth to Grand Portage. They are best shown just west of the Cook County–Lake County line, in Sec. 1, T. 58 N., R. 3 W., and in the southwest corner of Sec. 19, T. 60 N., R. 3 W.

GEOLOGIC FEATURES

In general the clastic stringers are a reddish argillaceous sandstone of fine to medium grain made up largely of the detritus of the near-by basaltic flows. Some of the grains are well rounded, and microscopic examination shows that they are largely plagioclase and augite, with minor amounts of magnetite or ilmenite. There is some quartz, and there are some fragments of basalt and earlier clastics. The secondary minerals, leucoxene, hematite, limonite, chlorite, and kaolinite, are common; epidote and calcite occur in some thin sections. The plagioclase is principally labradorite, as determined by the maximum symmetrical extinction angle of the albite twinning bands. Hematite gives the fillings a red color, and the kaolinite present in the matrix indicates some decomposition of the feldspars. Microscopic examination of the sandstone beds between lava flows showed the same mineral assemblages and similar oxidation and decomposition as the crevice fillings.

Several beds of tuff were found interbedded with the flows and

[5] "Keweenawan Series of Michigan," *Mich. Geol. Surv. Pub. 4* (geol. ser. 6, 1909), p. 31.

[6] "Section across the Keweenawan Lavas at Duluth, Minnesota," *Bull. Geol. Soc. Amer.*, Vol. XLIX (1938), pp. 809–10.

[7] "The Geology of the Anorthosites of the Minnesota Coast of Lake Superior," *Minn. Geol. Surv. Bull. 28* (1939), p. 31.

some fissures running down in the rocks below these beds were filled with tuffaceous material. Grout[8] reports that a thin section of a similar tuff shows glass shards with what appeared to be analcite cement. These tuffaceous fissure fillings commonly show in outcrops a patchwork of red and green blotches. A thin section showed the alteration of the original grains. The red material proved to be an ag-

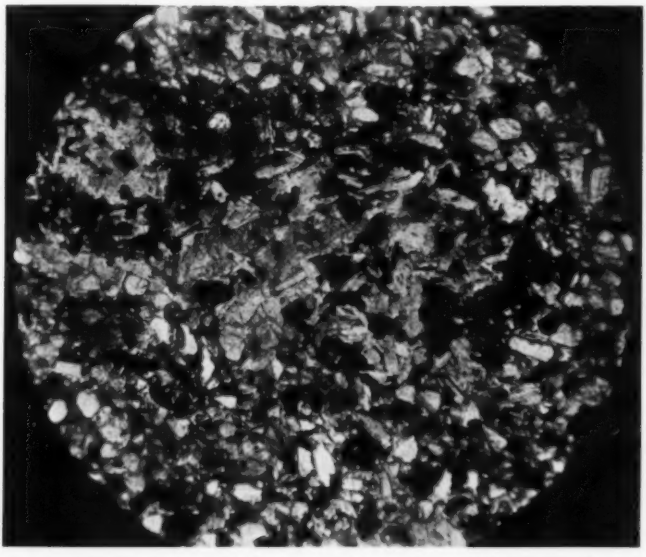

FIG. 3.—Photomicrograph of clastic material showing typical texture, also a fragment of an ophitic basalt flow.

gregate of basaltic glass fragments which was weathered and oxidized to a reddish porous mass surrounding patches of pale green, unaltered tuff. The beds of tuff dip in the same direction as the lava flows and probably result from deposition by water as does the bedding of the clastic stringers.

The contacts between the wall rocks and the sandstone fillings are sharp and give the appearance of having been slightly weathered before the material was washed into the cracks. Evidently the clas-

[8] *Ibid.*, p. 31.

tic material came from above, and none of the hundreds seen indicated a source below. The normal bedding of the clastic dikes indicates that they were laid down by water, and since the bedding dips at the same degree as the lava flow, the fillings must antedate the tilting.

The fissure fillings range in size from a hairline to six inches or more in width. On some flow outcrops they could be traced for sev-

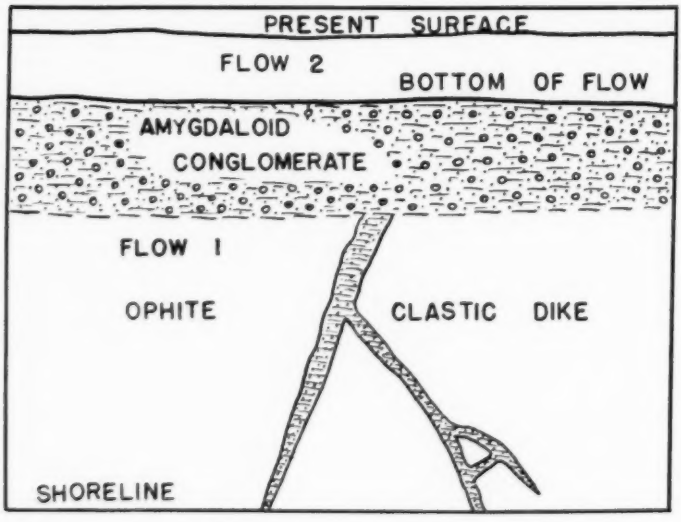

FIG. 4.—Cross section of flows showing relations of crevice fillings to amygdaloid conglomerate, and pinching out at depth.

eral hundreds of feet, but because of their branching and sinuous nature they are hard to trace for longer distances. Most of the stringers are in the upper eight or ten feet of the flows, but a large one may extend down several times this distance.

No regular pattern of the crevice fillings could be distinguished, but, rather, they are strikingly irregular in their surface pattern. They strike across the flows in every conceivable direction and generally have a nearly vertical dip, whereas the flows dip about fifteen to seventeen to the southeast. This is shown in a field sketch of part

of the wave-washed surface of a flow. A similar pattern in the McCarty's flow of New Mexico has been sketched by R. L. Nichols[9] and is here reproduced for comparison. The fissures seem to have been caused by thermal contraction rather than stretching or other causes. Their haphazard irregularity does not indicate response to any regional stresses or to movement of the flow while cooling. This irregu-

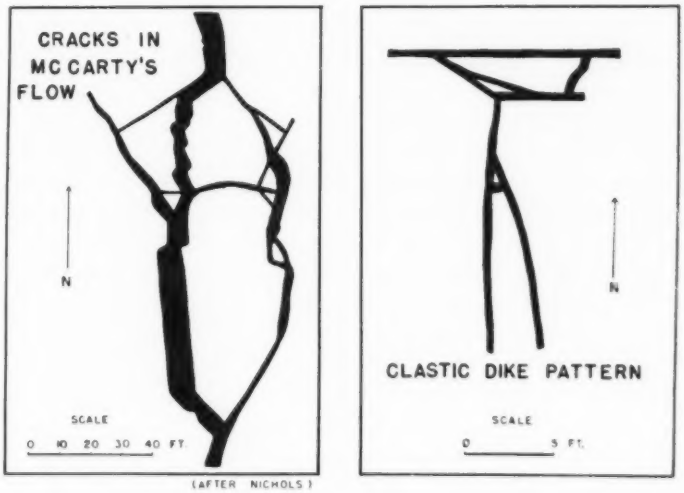

FIG. 5.—Comparison of cracks in McCarty's flow and dike pattern in Keweenawan flow. (By permission from Robert L. Nichols, "Surficial Banding and Shark's Tooth Projections in the Cracks of Basaltic Lava," *American Journal of Science*, Vol. CCXXXVII [1939].)

larity has been observed in other regions. M. G. Hoffman[10] reports that many of the Columbia River lavas are not columnar jointed but massive and broken by widely or closely spaced joints and in some cases irergularly fractured throughout the flow. A. E. Jones[11] dis-

[9] "Banding and Projections in Cracks in Basaltic Lavas," *Amer. Jour. Sci.*, Vol. CCXXXVII (1939), fig. facing p. 192.

[10] "Structural Features in the Columbia River Lavas of Central Washington," *Jour. Geol.*, Vol. XLI (1933), pp. 184–95.

[11] "Formation of Basaltic Lava Flows," *Proc. Geol. Soc. Amer.* (abstr., 1936) pp. 334–35.

tinguished three varieties of texture in the recent flows of Mount Lassen and in those of the Hawaiian Islands: (1) broken scoria, (2) black lava which broke cleanly while cooling, and (3) lava with a relatively smooth surface. If detrital materials filled openings in either of the first two, they would be crevice fillings resembling the Minnesota clastic dikes.

ORIGIN

The study of the nature of the fillings and their relationship to the adjacent rocks indicates that, during the cooling of the basaltic flow, numerous cracks and fissures were formed which decreased in size with depth. The normal processes of weathering and erosion converted the upper part of the flow into sediment which was washed down into the fissures. Volcanic tuffs were mixed with the ordinary clastic material in some places. Since many of the flows were of the "aa" type, the tops were blocky and afforded an easy start for weathering and erosion. Many of these blocky tops are represented today by amygdaloid conglomerates which have fine clastic material between blocks. This grades into the crevice fillings wedging out below the conglomerates. After these events the surface was covered by a subsequent flow or flows and the sequence was repeated. The fillings are older than the flows which overlie them. In at least one instance they were sufficiently consolidated to contribute material to a later conglomerate. The clastic dikes are cut by many veins of zeolites that seem clearly Keweenawan, and some are cemented by zeolitic material. This is further indication of their early origin and solidification.

ACKNOWLEDGMENTS.—My thanks are due to Dr. F. F. Grout and the Minnesota Geological Survey for the help and co-operation that were given me. I also wish to thank Dr. J. L. Rich for his critical readings and many helpful suggestions.

REVIEWS

Geochronologia Suecia Principles. By GERARD DE GEER. Stockholm: Almquist & Wiksells, 1940. Pp. 360; pls. 90.

This volume represents the summary of De Geer's long and important life's work. He defines geochronology as an exact chronology based on direct count, without any subjective assumptions, of annually deposited layers called "varves." By this technique it has been possible to date events back for at least 15,000 years. The duration of the Postglacial is 8,700 years, the Finiglacial 1,073 years, and the Gotiglacial 6,379 years. The standard chronology has been measured in Sweden, for which De Geer proposes the name "Standard Swedish Time Scale." This time scale has been correlated with our present calendar through the standard tree-ring chronology. De Geer expresses no concern over the validity of the details of this latter chronology, for it is not of particular concern to his dates. In his varve chronology he has selected as the zero year a particular varve which was formed when the ice-dammed Lake Storsjön was drained. This event marked the first definitive bipartition of the land-ice (Pl. 63) and so is chosen as the end of the Late Glacial and the beginning of the Postglacial. This event took place 5690 B.C. within an error of only a few years.

Varve measurements from North America, Argentina, Central Africa, India, and New Zealand have been successfully and definitely correlated with the Standard Swedish Time Scale by what is called "teleconnection." "Detailed similarity is there, quite as striking between most widely separated as between neighboring localities and definitely certifying that it must depend on a universal cause." The varve record is the registration of solar radiation and must be congruent all over the earth when not obscured by local deviations.

From a small tentative beginning, 1878–1905, the work grew until between 1905 and 1915 the Swedish chronology was successfully established. From 1921 to date the teleconnections have been established.

A varve is the whole layer of one year's deposition, having a coarser-grained part deposited in summer and a finer-grained (usually clay) part deposited in the winter. They are formed only in fresh or slightly salty water. Near the ice border the varves are thick. Farther away they be-

come thinner, until at considerable distance they are so thin that they have to be measured under a magnifying glass. De Geer calls them respectively proximal, distal, and microdistal varves.

A chapter on the "Technique of Measuring Varves" gives many detailed directions and points out possible sources of error. This discussion is very important to the whole science of geochronology, for only by an understanding of the irregularities of the record and their correct interpretation can correlation be achieved. The reviewer feels that the choice of the word "error" is here unfortunate, for what De Geer has in mind is interpretation or misinterpretation. He gives twelve types of "physical errata" such as "changed distance from receding river-mouth" etc., and six types of "observational errata" such as "deceptive bands, simulating winter-layers: *digraphs*, *trigraphs*," etc. It is inferred from the book, though the author does not say so, that decision in such a case as the latter must be left open until the definite parts of the record, above and below the doubtful place, have been correlated with either the Standard Swedish Scale or with some other nearby measurement which can be correlated and where the doubtful part of the first record is clear. Then the doubtful varves may be interpreted and the first measured section either corrected or the whole remeasured in light of the new interpretation (see Plate 45, column 5). When this has been successfully accomplished the local and chronologically unimportant variations are eliminated, and the normal or world-wide variations are left. When varves are put into a graph, a heavy line is used when it is found that the record is of varves whose thicknesses respond to the world-wide causes, whereas a thin line is used for varve thicknesses which have resulted from some local cause and so have no meaning in world-chronology.

When the varves alternate in thickness in successive years it is called a "biennic variation." This sawtooth pattern of the graph thus produced during a series of years furnishes a convenient method of comparing unknown curves. By plotting the patterns of biennic variations De Geer finds characteristic "constellations" by the use of which the long and tedious labor of matching unknown graphs is vastly simplified. Furthermore, by comparing the constellations of biennic variations as shown on Plate 90 the average reader can see much more significance to the correlations than he can by looking at the original curves.

A full discussion of the Swedish eskers ("oses") follows in the next chapter. They are shown to be composed of a continuous string of "ose-centers" deposited where glacial rivers emerged at the ice edge into the standing water of the great Baltic ice lake. Each ose-center is shown to

be the result of one year's deposition of coarse material. Each has been traced into and correlated with a particular varve formed farther out in the lake. In this way the ose deposition has been precisely dated. The oses were not formed by contemporaneous deposition all along the channel of a subglacial river but were made as a succession of ose-centers which have been somewhat softened and joined by wave-washed material during emergence from the waters of the lake.

The next chapter presents a detailed account of the Stockholm region (the region in which varve chronology was first worked out). Annual frontal moraines, eskers, and varves are described. The ice edge receded rather uniformly and regularly across this region at an average rate of 120–200 meters per year. The year −1073 before zero is taken as the line between Gotiglacial and Finiglacial—the year in which the great Baltic lake fell to sea-level and marine waters and fauna entered somewhat into the southwest part. The ice edge stood just south of Stockholm at this time. Special features of the area are the so-called "seismic moraines" composed of blocks of local bedrock whose origin is convincingly attributed to local rupturing of the bedrock by seismic activity, which occurred −1050 b.Z. or −9690 before 1900 A.D. The area in which these seismic annual moraines or "mora" (boulder pile) occur is limited to a belt 12 km. long and 2 or 3 km. wide. Another similar mora of annual moraines is found at Aras Bay, Lake Venern. Annual moraines in general are best explained as being subaqueous features where the ice-foot is sharp, so that winter pushing plows up these ridges.

The next chapter deals with the Upsala region. Then follows a description of measuring the standard varve line in 1905. It was divided into sections, each one of which was measured by a student, and then the whole assembled and correlated. The recession discovered was normally between 200 and 400 meters per year, but it was also discovered that over the Botnic Valley between years −500 to −600 b.Z. calving caused recession of 2,000 meters per year. This resulted in notable change in direction of the ice margin as well as a great deal of ice-rafted till in the record of this time. The long varve records of the Angeman Valley, described by Lidén, furnish a basis of measuring the rate of land upheaval during the last 9,000 years—beginning with a maximum of 15 meters per century and falling to 1 meter per century at present. The graph of this rate—Figure 43—is a significant contribution to tectonic geology.

The next chapter describes the varve expeditions to other continents. The expeditions of 1920 to North America, of 1924–25 to northwest Himalaya, of 1925–29 to Argentina, of 1927–28 to Central East Africa,

and of 1932–34 to New Zealand, all yielded measurements correlated and dated with the standard Swedish chronology.

In conclusion the author expresses the mature conviction that variations in varve thickness are related directly to solar variations. He, however, finds no periodic variation commensurate with the sun-spot cycle which seems to have affected the trees but not the varves.

The last hundred pages are devoted to description of tables, of symbols, the "Main Time Scale," of maps and plates in the accompanying Atlas, and to an extensive Bibliography. The book is written in excellent English with just enough idiomatic mistakes to make it delightful reading. It is slightly repetitious in places, but this may serve a good purpose for clarity and emphasis. The presentation of each subject is given in chronological arrangement, telling how the idea or technique evolved from its beginning in 1884 to its present status. It is illustrated with excellent photographs, three of which are in color.

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"Relation of Thickness of Mississippian Limestones in Central and Eastern Kansas to Oil and Gas Deposits," by WALLACE LEE. (*Geological Survey Bulletin 26.*) Lawrence, Kan., 1939. Pp. 42; figs. 4; pls. 3.

The data upon which this report was based were obtained by the study of the logs and in some cases cuttings from about 3,500 oil and gas wells. The thickness of the Mississippian limestones is shown on an accompanying map, together with the close relation of the thickness to some of the structural features and to the oil and gas fields where these limestones are the producing horizons. Pre-Pennsylvanian folding and peneplanation were responsible for the removal of the Mississippian completely over large areas and in other areas for great thinning on the structural highs. These older anticlines can be traced by the thinning of the Mississippian limestone, but the oil and gas accumulations as indicated by this study are due not so much to the structures as to the porosity developed by weathering and leaching on the pre-Pennsylvanian erosion surface.

A list of fields that have produced from Mississippian rocks, and the available production figures, are included.

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